Chapter 14
Statistical Methods for Quality Control

Philosophies and Frameworks

- **Quality**
  - features and characteristics of a product or service that bears on its ability to satisfy given needs

- **Total Quality**
  - a people-focused management system aimed at continual increase in customer satisfaction at continually lower real cost

- **Founding Fathers**
  - Deming
    - advocated that managers develop a culture of commitment to quality
  - Juran
    - quality = fitness for use: quality planning, quality control, quality improvement
  - Common ground
    - management involvement
    - continuous improvement
    - importance of training
    - need to use Quality Control techniques

Philosophies and Frameworks

- **Baldrige National Quality Award (1987)**
  - created to raise awareness and to recognize US organizations

- **ISO 9000 (1987)**
  - international standards defining what is needed to maintain an efficient quality conformance system
    - eg: measuring system calibration, record keeping systems

- **Six Sigma (late 80's)**
  - Motorola's goal: quality at rate of 3.4 dpmo
  - relies heavily on statistics

  - Quality Engineering
    - consider quality when designing products and processes and identify quality problems prior to production

  - Quality Control
    - inspections/measurements to determine if quality standards are being met
Statistical Process Control

- Classical Hypothesis Testing generally involves making a one-time decision based on a single sample
- Statistical Process Control
  - many situations can change over time and must be monitored on an ongoing basis
  - why take frequent, periodic samples?

2 Sources of Process Variation

- Assignable Cause Variation
  - nonrandom influences arising out of factors that can be corrected
  - e.g., wear & tear on machinery, operator error, contaminated materials
  - signals that the process is out of control
  - attempt to identify the underlying cause and take corrective action
  - how evidenced on control chart?
- Common Cause (Chance) Variation
  - random factors that cannot be avoided/corrected
  - are unavoidable even when the process is in control
  - e.g., temperature changes, slight variations in materials
  - requires changing the process itself in order to reduce/eliminate
    - e.g., invest in new machinery, better training, more supervision, new suppliers
- Driving analogy

Quality Control Inference

- $H_0$: The process is in control
  - i.e., no sources of assignable cause variation are present
- $H_A$: The process is out of control
  - i.e., sources of assignable cause variation are present

- Type I error?
- Type II error?
Control Charts

- Graphically depict process performance over time
- Constructed after a process has stabilized and is working as it is designed to
- Used to monitor variation in an important characteristic that affects quality of the product
- Control Chart has
  - Centerline
  - Upper Control Limit
  - Lower Control Limit
  - Data points for each sampled time period
  - Each time period is a hypothesis test of the process being in control
- Aids in the detection of assignable cause variation
  - How?

- Control limits are set at $CL \pm 3(\text{standard errors})$
  - by setting control limits at $\pm 3$ std errors, $\alpha = \ldots$
  - why?

- $\bar{x}$ Chart: Process Mean and Standard Deviation Known
  - Tracks the value of the sample mean around the process' mean when the process is in control
  - Centerline set at $\mu$

  Control limits = $\mu \pm 3\sigma_x$
**X Chart:** Process Mean and Standard Deviation Unknown

- X vs. X
- Tracks the values of the sample mean around grand mean
- Centerline set at \( \bar{X} \)
- Control Limits: \( UCL = \bar{X} + A_2 \bar{R} \)
- Note: limits for X chart depend on sample ranges
- Acme Pipe assignment

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**R-Chart**

- Tracks process variation over time
- Higher variation suggests items tend to be dissimilar and of inconsistent quality
- Standard deviation is a more comprehensive measure, but:
  - range is easier to compute
  - range is more intuitive
  - not much information is lost when \( n \) is small (\( n<10 \))
- Centerline set at \( R \)
- Control Limits: \( UCL_R = D_4 \bar{R} \), \( LCL_R = D_3 \bar{R} \)
- Acme Pipe assignment
- Since X chart uses the average range, an R-chart is usually constructed first; only interpret/use X chart if R-chart indicates that process variation is in control

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**Preliminary vs Revised Control Charts**

- Any initial sample falling outside the preliminary control limits should be investigated
- If assignable cause variation is identified, those points should be dropped and the control limits should be recalculated using the remaining samples
- Use the revised limits for monitoring the process in the future
Interpretation Of Control Charts

- A process is judged to be in control when:
  - all sample data points lie within UCL and LCL and
  - only random variation is present
- even if all sample points fall within the three sigma limits, the process may not be in control
- nonrandom patterns or trends can indicate a change in the process and a loss of control
- examples of nonrandom patterns:
  - something other than 68% of points within \( \pm 1 \) SD of centerline
  - something other than 95% of points within \( \pm 2 \) SD of centerline
  - 9 points in a row on one side of a centerline
  - 6 points in a row that are consistently increasing or decreasing

p Chart

- Applicable when a unit of product is judged as being either acceptable or not acceptable
- Tracks the proportion of defective items over time
- \( \hat{p} \) vs. \( \hat{p} \)
- When \( p \) is unknown, pool all samples to estimate it
- Centerline set at \( p \)
- Control limits:
  - \( \hat{p} \pm 3 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \)
  - if LCL < 0, set it at 0 since it is impossible to have fewer than 0% defectives
- Microchip assignment

Acceptance Sampling

- Sample items from a lot to infer whether the entire lot meets specifications
- Based on the number of defective items in the sample, either accept or reject the entire lot
- Advantages over 100% inspection:
  - less expensive
  - less product damage
  - fewer inspectors required
  - only option when testing is destructive
  - inspector fatigue (nib)
Acceptance Sampling

- **H₀**: _______________ vs **Hₐ**: _______________

- **Type I error**
  - Reject a good-quality lot
  - Producer’s risk = P(reject lot | good-quality lot) = α

- **Type II error**
  - Accept a poor-quality lot
  - Consumer’s risk = P(accept lot | poor-quality lot) = β

- **Acceptance Criterion (c)**
  - Maximum number of defects allowed in sample without rejecting the lot
  - Ensures that the AQL is maintained without rejecting more than a prescribed percentage of acceptable lots

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Operating Characteristic Curve

- Illustrates how p, c, n affect the probability of accepting a lot

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Selecting an Acceptance Sampling Plan

- In formulating a plan, managers specify two values for the proportion of defectives in the lot
  - **Producer’s Risk**
    - α = the probability that a lot with p₀ defectives will be rejected
  - **Consumer’s Risk**
    - β = the probability that a lot with p₁ defectives will be accepted

- Then, the values of n and c are selected that result in an acceptance sampling plan that comes closest to meeting both the α and β requirements specified
Operating Characteristic Curve

Selecting an Acceptance Sampling Plan: A Special Case

- Special case: $p_0 = p_1$
- Determine $c$ by using Cumulative Binomial tables
  - select column for $p_0$, $p_1$
  - locate $c$ as the cumulative probability exceeding (1 - Producer's Risk)
  - if more than $c$ defects are found in the sample, reject the lot

$$P_B(x \leq c \mid n, p) \geq (1 - \text{Producer's Risk})$$

Multi-Stage Sampling Designs

- Rather than using a single sample to determine whether to accept a lot
- Take a preliminary sample... then, depending on its result either: (1) accept the lot, (2) reject the lot, or (3) take another sample, pooling it with previous samples, and then decide
- Is generally more efficient than a single-sampling design
  - i.e., a smaller total sample size to inspect/test