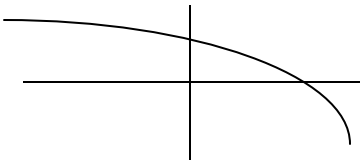
Intermediate Algebra – Final Exam Review

ANSWER KEY

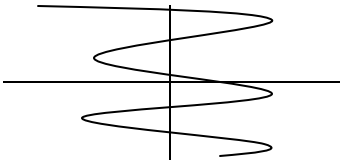
GENERAL FUNCTION UNDERSTANDING

1. Answers vary. Samples shown below.

a) A graph of a decreasing function



b) A graph that does NOT represent a function



2. $f(10) = 65$ (10, 65) and $f(-10) = 105$ (-10, 105)

3. $f(-1) = 3$ (-1,3)

4. a. $k(3) = 5$ (3,5) b. $k(2) = 17$ (2,17)
c. Domain: {-1, 2, 3, 8, 17} Range: {0, 3, 5, 17, 62}

5. a. $f(0) = 4$ b. x = -1 and 1 c. Domain: $-3 \le x < 2$ [-3,2) Range: $-5 \le f(x) \le 4$ [-5,4]

6. a. $f(2) + g(2) =$ -18 b. $f(x) - g(x) = x^2 - 3x - 10$ c. $f(x) \cdot g(x) = x^3 - 7x^2 - 5x + 75$
d. $g(f(2)) =$ -20 e. $g(g(2)) =$ -8 f. $f(g(2)) = 0$
g. $f(g(x)) = x^2 - 12x + 20$ h. $g(f(x)) = x^2 - 2x - 20$ i. $g(g(x)) = x - 10$

7. a. C(x) = 3.50x + 12
b. It costs \$82 to tow the car 20 miles.
c. C(8) = 40 (8, 40) It costs \$40 to tow the car 8 miles.
d. $x \approx 25.14$ (25.14,100) It costs \$100 to tow the car 25.14 miles.
e. C(15) = 64.5 It costs \$64.50 to tow the car 15 miles.
f. Practical domain of $C(x)$: $0 \le x \le 30$ miles
g. Practical range of $C(x)$: $\$12 \le C(x) \le \$117$

LINEAR FUNCTIONS

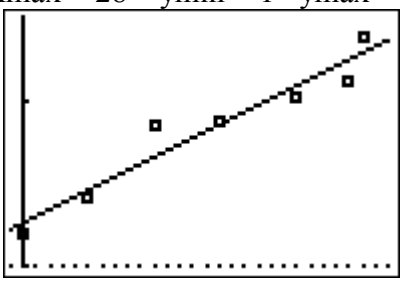
8. $y = \frac{5}{8}x + \frac{31}{8}$

9. $y = 1$

10. $x = 3$

11. a. (0,118.4) After 0 months, the value of this investment is \$118,400.
b. (51.3, 0) After about 51.3 months, the investment will be worth \$0.
c. Slope = -2.31 The value of this investment is decreasing at a rate of \$2,310 per year.

12. a. Average rate of change between 1985 and 2005 = 0.036 million dollars per year
From 1985 to 2005, the total sales increased at an average rate of \$36,000 per year.
b. Not a linear function. The rate of change is not constant.
c. $S(t) = 0.0394t + 1.2594$
d. Graph shown with Xmin = -1 Xmax = 28 ymin = 1 ymax = 2.5



e. t = 28, S(28) = 2.36 million dollars (\$2,360,000)
f. t = 44.18 Sales will reach \$3,000,000 in the year 2024

EXPONENTIAL FUNCTIONS

13. $p(x) = 52 - 11x$ $g(x) = 128(0.5)^x$ $h(x) = 1000(1.1)^x$

14. a. $N(t) = 500 + 20t$ b. $N(t) = 500(1.02)^t$ c. $N(t) = 500 - 30t$ d. $N(t) = 500(0.7)^t$
e. $N(t) = 500(2)^t$ f. $N(t) = 500$

15. a. x = 132.87 b. x = -16.35

16. a. Growing by 15% per year b. 300,000 people c. 3,712,636 people d. 2004

17. a. (0,142) b. DNE c. $-\infty<x<\infty$ $(-\infty, \infty)$ d. $D(x)>0$ $(0,\infty)$ e. the line y=0

18. a. $V(t) = 30000(0.92)^t$ b. 4.9 years c. 8.3 years

19. a. $P(t) = 26619(0.9932)^t$ b. Declining at a rate of 0.68% per year c. 18290 people
d. x=41.9 The population reached 20000 at the end of the year 1971

LOGARITHMIC FUNCTIONS

20. $\log_5(1) = 0$ $\log_5(5) = 1$ $\log_5(125) = 3$ $\log_5(0)$ DNE
$\log_5\left(\frac{1}{5}\right) = -1$ $\log_5\left(\frac{1}{25}\right) = -2$ $\log_5\left(\sqrt{5}\right) = 1/2$

21. $\log_4(64) = 3$

22. $6^2 = 36$ $10^3 = 1000$

23. $\log_5 640 = \dfrac{\log(640)}{\log(5)} = 4.01$. Round to the nearest hundredth.

24. $30 - 5\log_2 8 = 15$

25. a. DNE b. (1,0) c. Domain: x>0 (0,∞) d. Range: -∞ < f(x) < ∞ (-∞, ∞)
e. the line x = 0 f. f(25) = 4.644 g. x = 8

26. x = 111

27. $x = \log_{1.15}(2) \approx 4.959$

28. a. m(120) = 19.61 b. d = 43.11 inches

QUADRATIC FUNCTIONS

29. $12x^2 + 4x = 4x(3x+1)$ $x^2 + 4x - 5 = (x+5)(x-1)$ $x^2 - 36 = (x+6)(x-6)$

30. x = -7, x = 3

31. a. Downward b. (0,11) c. (-1.16,0) and (3.16,0) d. (1,14) e. x = 1
f. Domain: -∞<x<∞ (-∞, ∞) g. Range: f(x) ≤ 14 (-∞, 14]

32. Exact solution: $x = -\dfrac{3}{8} + \dfrac{\sqrt{7}}{8}i$ and $x = -\dfrac{3}{8} - \dfrac{\sqrt{7}}{8}i$
Approximate solution: $x = -0.38 + 0.33i$ and $x = -0.38 - 0.33i$

33.
a. After 2 seconds, the ball is 96 feet above the ground.
b. t = 3.5 seconds. The ball is 24 feet above the ground 3.5 seconds after being thrown from the roof. NOTE: You may have also found t = -1 to be a solution, but this does not make sense in the context of this problem (it is not in the practical domain of this function).
c. $-16t^2 + 40t + 80 = 0$ t ≈ 3.81 seconds
d. Vertex: (1.25,105) The maximum height of the ball is 105 feet.

34. $\sqrt{-9} = 3i$ $\sqrt{-11} = i\sqrt{11} \approx 3.32i$ $\sqrt{-18} = 3i\sqrt{2} \approx 4.24i$

$3 - \sqrt{-16} = 3 - 4i$ $\dfrac{4 + \sqrt{-3}}{6} = \dfrac{2}{3} + \dfrac{\sqrt{3}}{6}i \approx 0.67 + 0.29i$

35. $i^2 = -1$ $-i^2 = 1$ $3i(5-2i) = 6 + 15i$ $(3+i) - (2-3i) = 1 + 4i$ $(3+i)(2-3i) = 9 - 7i$

RATIONAL FUNCTIONS

36. $f(x)=\dfrac{4x+1}{5x^2}$ Domain: All real numbers except 0 Vertical Asymptote: the line x = 0

$f(x)=\dfrac{x^2+4x-6}{x+3}$ Domain: All real numbers except -3 Vertical Asymptote: the line x = -3

$f(x)=\dfrac{5x-1}{2x+8}$ Domain: All real numbers except -4 Vertical Asymptote: the line x = -4

37. $f(x)=\dfrac{4x+1}{5x^2}$ Horizontal Asymptote: the line y = 0

$f(x)=\dfrac{x^2+4x-6}{x+3}$ No Horizontal Asymptote

$f(x)=\dfrac{5x-1}{2x+8}$ Horizontal Asymptote: the line y = 5/2

38. $x=-\dfrac{5}{3}$.

39. a. (0,-4) b. (4/3,0) c. All real numbers except 1 d. the line x = 1
e. the line y = -3 f. f(5) = -11/4 = -2.75 g. x = 7/6

RADICAL FUNCTIONS

40. $\sqrt{32}$ =5.66 $\sqrt[4]{42}$ = 2.55 $\sqrt[3]{-30}$ = -3.11 $\sqrt[6]{-36}$ DNE

41. $f(x)=\sqrt{6-2x}$ Domain: $x \le 3$ $(-\infty,3]$

$f(x)=\sqrt[3]{2x+1}$ Domain: $-\infty < x < \infty$ $(-\infty, \infty)$

$f(x)=\sqrt[4]{x+7}$ Domain: $x \ge -7$ $[-7,\infty)$

42. a. Domain: $x \ge -4$ $[-4,\infty)$ b. $\left(0,\sqrt{8}\right)$ or $\left(0,2\sqrt{2}\right)$ or (0,2.828) c. (-4,0)
d. $f(5) = \sqrt{18} = 3\sqrt{2} \approx 4.243$ e. x = 1/2.

43. x = 3/2 = 1.5

44. x = 3

45. x = -125/3 ≈ -41.67