

Lesson 9 – Solving Quadratic Equations

We will continue our work with Quadratic Functions in this lesson and will learn several methods for solving quadratic equations.

Graphing is the first method you will work with to solve quadratic equations followed by factoring and then the quadratic formula. You will get a tiny taste of something called Complex Numbers and then will finish up by putting all the solution methods together.

Pay special attention to the problems you are working with and details such as signs and coefficients of variable terms. Extra attention to detail will pay off in this lesson.

Lesson Topics

Section 9.1: Quadratic Equations in Standard Form

- Horizontal Intercepts
- Number and Types of solutions to quadratic equations

Section 9.2: Factoring Quadratic Expressions

- Factoring using the method of Greatest Common Factor (GCF)
- Factoring by Trial and Error

Section 9.3: Solving Quadratic Equations by Factoring

Section 9.4: The Quadratic Formula

Section 9.5: Complex Numbers

Section 9.6: Complex Solutions to Quadratic Equations

Lesson 9 Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Mini-Lesson 9

Section 9.1 – Quadratic Equations in Standard Form

A QUADRATIC FUNCTION is a function of the form

$$f(x) = ax^2 + bx + c$$

A QUADRATIC EQUATION in STANDARD FORM is an equation of the form

$$ax^2 + bx + c = 0$$

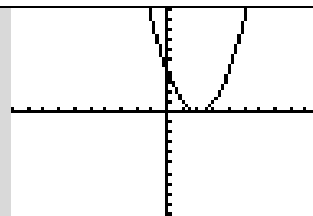
If the quadratic equation $ax^2 + bx + c = 0$ has real number solutions x_1 and x_2 , then the x -intercepts of $f(x) = ax^2 + bx + c$ are $(x_1, 0)$ and $(x_2, 0)$.

Note that if a parabola does not cross the x -axis, then its solutions lie in the complex number system and we say that it has *no real x -intercepts*.

There are three possible cases for the number of solutions to a quadratic equation in standard form.

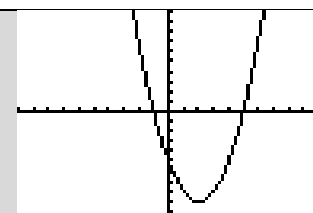
CASE 1: One, repeated, real number solution

The parabola touches the x -axis in *just one* location
(i.e. only the vertex touches the x -axis)



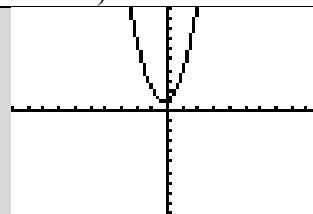
CASE 2: Two unique, real number solutions

The parabola crosses the x -axis at
two unique locations.



CASE 3: No real number solutions (but two Complex number solutions)

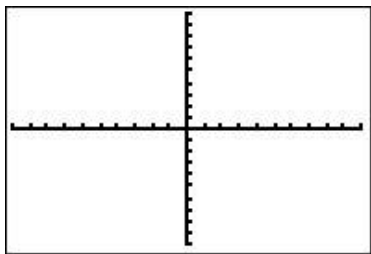
The parabola does NOT cross the x -axis.



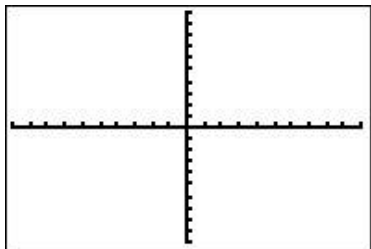
Problem 1 **MEDIA EXAMPLE – HOW MANY AND WHAT KIND OF SOLUTIONS?**

Use your graphing calculator to help you determine the number and type of solutions to each of the quadratic equations below. Begin by putting the equations into standard form. Draw an accurate sketch of the parabola in an appropriate viewing window. If your solutions are real number solutions, use the graphing INTERSECT method to find them. Use proper notation to write the solutions and the horizontal intercepts of the parabola. Label the intercepts on your graph.

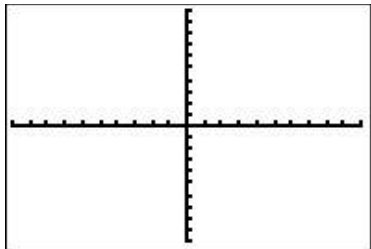
a) $x^2 - 10x + 25 = 0$



b) $-2x^2 + 8x - 3 = 0$



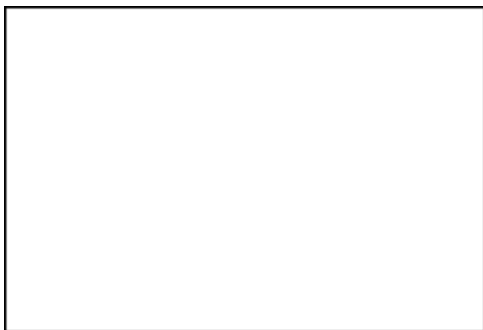
c) $3x^2 - 2x = -5$



Problem 2 | YOU TRY – HOW MANY AND WHAT KIND OF SOLUTIONS?

Use your graphing calculator to help you determine the number and type of solutions to each of the quadratic equations below. Begin by putting the equations into standard form. Draw an accurate sketch of the parabola in an appropriate viewing window (the vertex, vertical intercept, and any horizontal intercepts should appear on the screen). IF your solutions are real number solutions, use the graphing INTERSECT method to find them. Use proper notation to write the solutions and the horizontal intercepts of the parabola. Label the intercepts on your graph.

a) $-x^2 - 6x - 9 = 0$



Xmin = _____ Ymin = _____

Xmax = _____ Ymax = _____

Number of Real Solutions: _____

Real Solutions: _____

b) $3x^2 + 5x + 20 = 0$



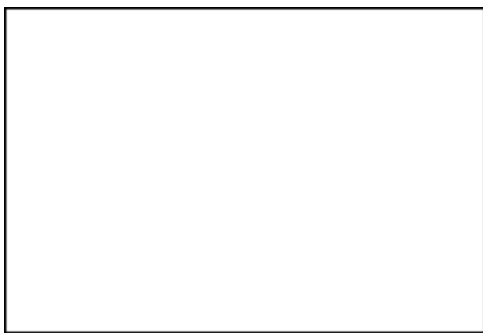
Xmin = _____ Ymin = _____

Xmax = _____ Ymax = _____

Number of Real Solutions: _____

Real Solutions: _____

c) $2x^2 + 5x = 7$



Xmin = _____ Ymin = _____

Xmax = _____ Ymax = _____

Number of Real Solutions: _____

Real Solutions: _____

Section 9.2 –Factoring Quadratic Expressions

So far, we have only used our graphing calculators to solve quadratic equations utilizing the Intersection process. There are other methods to solve quadratic equations. The first method we will discuss is the method of FACTORING. Before we jump into this process, you need to have some concept of what it means to FACTOR using numbers that are more familiar.

Factoring Whole Numbers

To FACTOR the number 60, you could write down a variety of responses some of which are below:

- $60 = 1 \cdot 60$ (not very interesting but true)
- $60 = 2 \cdot 30$
- $60 = 3 \cdot 20$
- $60 = 4 \cdot 3 \cdot 5$

All of these are called FACTORIZATIONS of 60, meaning to write 60 as a product of some of the numbers that divide it evenly.

The most basic factorization of 60 is as a product of its prime factors (remember that prime numbers are only divisible by themselves and 1). The PRIME FACTORIZATION of 60 is:

$$60 = 2 \cdot 2 \cdot 3 \cdot 5$$

There is only one PRIME FACTORIZATION of 60 so we can now say that 60 is COMPLETELY FACTORED when we write it as $60 = 2 \cdot 2 \cdot 3 \cdot 5$.

When we factor polynomial expressions, we use a similar process. For example, to factor the expression $24x^2$, we would first find the prime factorization of 24 and then factor x^2 .

$$24 = 2 \cdot 2 \cdot 2 \cdot 3 \quad \text{and} \quad x^2 = x \cdot x$$

Putting these factorizations together, we obtain the following:

$$24x^2 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x$$

Let's see how the information above helps us to factor more complicated polynomial expressions and ultimately leads us to a second solution method for quadratic equations.

Problem 3 | **WORKED EXAMPLE – Factoring Using GCF Method**

Factor $3x^2 + 6x$. Write your answer in completely factored form.

The building blocks of $3x^2 + 6x$ are the terms $3x^2$ and $6x$. Each is written in FACTORED FORM below.

$$3x^2 = 3 \cdot x \cdot x \quad \text{and} \quad 6x = 3 \cdot 2 \cdot x$$

Let's rearrange these factorizations just slightly as follows:

$$3x^2 = (3 \cdot x) \cdot x \quad \text{and} \quad 6x = (3 \cdot x) \cdot 2$$

We can see that $(3 \cdot x) = 3x$ is a common FACTOR to both terms. In fact, $3x$ is the GREATEST COMMON FACTOR (GCF) to both terms.

Let's rewrite the full expression with the terms in factored form and see how that helps us factor the expression:

$$\begin{aligned} 3x^2 + 6x &= (\mathbf{3 \cdot x}) \cdot x + (\mathbf{3 \cdot x}) \cdot 2 \\ &= (\mathbf{3x}) \cdot x + (\mathbf{3x}) \cdot 2 \\ &= (\mathbf{3x})(x + 2) \\ &= \mathbf{3x(x + 2)} \end{aligned}$$

Always CHECK your factorization by multiplying the final result.

$$3x(x + 2) = 3x^2 + 6x \text{ CHECKS}$$

Problem 4 | **MEDIA EXAMPLE – Factoring Using GCF Method**

Factor the following quadratic expressions. Write your answers in completely factored form.

a) $11a^2 - 4a$

b) $55w^2 + 5w$

Problem 5 | **YOU TRY – Factoring Using GCF Method**

Factor the following quadratic expression. Write your answers in completely factored form.

a) $64b^2 - 16b$

b) $11c^2 + 7c$

If there is no common factor, then the GCF method cannot be used. Another method used to factor a quadratic expression is shown below.

Factoring a Quadratic Expressions of the form $x^2 + bx + c$ by TRIAL AND ERROR

$$x^2 + bx + c = (x + p)(x + q),$$

$$\text{where } b = p + q \text{ and } c = p \cdot q$$

Problem 6 | **WORKED EXAMPLE – Factoring Using Trial and Error**

Factor the quadratic expression $x^2 + 5x - 6$. Write your answer in completely factored form.

Step 1: Look to see if there is a common factor in this expression. If there is, then you can use the GCF method to factor out the common factor.

The expression $x^2 + 5x - 6$ has no common factors.

Step 2: For this problem, $b = 5$ and $c = -6$. We need to identify p and q . In this case, these will be two numbers whose product is -6 and sum is 5 . One way to do this is to *list* different numbers whose product is -6 , then see which pair has a sum of 5 .

Product = -6	Sum = 5
$-3 \cdot 2$	No
$3 \cdot -2$	No
$-1 \cdot 6$	YES
$1 \cdot -6$	No

Step 3: Write in factored form

$$x^2 + 5x - 6 = (x + (-1))(x + 6)$$

$$x^2 + 5x - 6 = (x - 1)(x + 6)$$

Step 4: Check by foiling.

$$\begin{aligned} (x - 1)(x + 6) &= x^2 + 6x - x - 6 \\ &= x^2 + 5x - 6 \quad \text{CHECKS!} \end{aligned}$$

Problem 7 **MEDIA EXAMPLE – Factoring Using Trial and Error**

Factor each of the following quadratic expressions. Write your answers in completely factored form. Check your answers.

a) $a^2 + 7a + 12$

b) $w^2 + w - 20$

c) $x^2 - 36$

Problem 8 **YOU TRY – Factoring Using Trial and Error**

Factor each of the following quadratic expressions. Write your answers in completely factored form. Check your answers.

a) $n^2 + 8n + 7$

b) $r^2 + 3r - 70$

c) $m^2 - m - 30$

Section 9.3 – Solving Quadratic Equations by Factoring

In this section, we will see how a quadratic equation written in standard form: $ax^2 + bx + c = 0$ can be solved *algebraically* using FACTORING methods.

The Zero Product Principle
If $a \cdot b = 0$, then $a = 0$ or $b = 0$

To solve a Quadratic Equation by FACTORING:

Step 1: Make sure the quadratic equation is in standard form: $ax^2 + bx + c = 0$

Step 2: Write the left side in Completely Factored Form

Step 4: Apply the ZERO PRODUCT PRINCIPLE

Set each linear factor equal to 0 and solve for x

Step 5: Verify result by graphing and finding the intersection point(s).

Problem 9 | **WORKED EXAMPLE–Solve Quadratic Equations By Factoring**

a) Solve by factoring: $5x^2 - 10x = 0$

Step 1: This quadratic equation is already in standard form.

Step 2: Check if there is a common factor, other than 1, for each term (yes... $5x$ is common to both terms)

Step 3: Write the left side in Completely Factored Form

$$5x^2 - 10x = 0$$

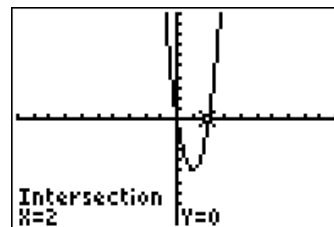
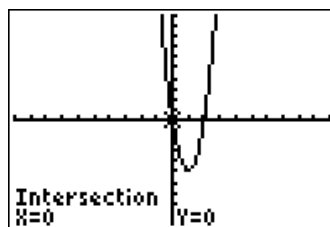
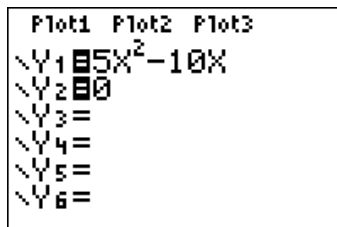
$$5x(x - 2) = 0$$

Step 4: Set each linear factor equal to 0 and solve for x :

$$5x = 0 \quad \text{OR} \quad x - 2 = 0$$

$$x = 0 \quad \text{OR} \quad x = 2$$

Step 5: Verify result by graphing.



b) Solve by factoring: $x^2 - 7x + 12 = 2$

Step 1: Make sure the quadratic is in standard form.

Subtract 2 from both sides to get: $x^2 - 7x + 10 = 0$

Step 2: Check if there is a common factor, other than 1, for each term.

Here, there is no common factor.

Step 3: Write the left side in Completely Factored Form

$$x^2 - 7x + 10 = 0$$

$$(x + (-5))(x + (-2)) = 0$$

$$(x - 5)(x - 2) = 0$$

Step 4: Set each linear factor to 0 and solve for x:

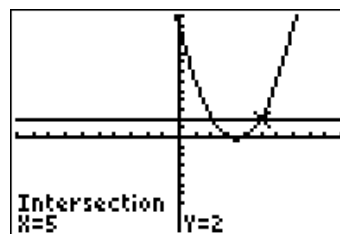
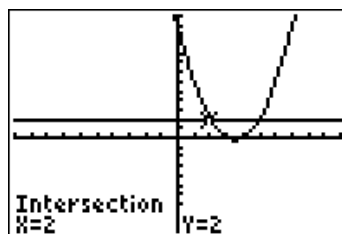
$$(x - 5) = 0 \quad \text{OR} \quad (x - 2) = 0$$

$$x = 5 \quad \text{OR} \quad x = 2$$

Step 5: Verify result by graphing (Let $Y1 = x^2 - 7x + 12$, $Y2 = 2$)

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Plot1 Plot2 Plot3
Y1=X^2-7X+12
Y2=2
Y3=
Y4=
Y5=
Y6=
  
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Problem 10**MEDIA EXAMPLE–Solve Quadratic Equations By Factoring**

Solve the equations below by factoring. Show all of your work. Verify your result by graphing.

a) Solve by factoring: $-2x^2 = 8x$



b) Solve by factoring: $x^2 = 3x + 28$



c) Solve by factoring: $x^2 + 5x = x - 3$



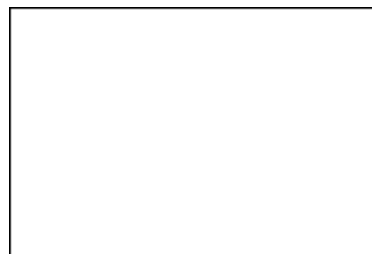
Problem 11 **YOU TRY – Solving Quadratic Equations by Factoring**

Use an appropriate factoring method to solve each of the quadratic equations below. Show all of your work. Be sure to write your final solutions using proper notation. Verify your answer by graphing. Sketch the graph on a good viewing window (the vertex, vertical intercept, and any horizontal intercepts should appear on the screen). **Mark and label** the solutions on your graph.

a) Solve $x^2 + 3x = 10$



b) Solve $3x^2 = 17x$



Section 9.4 –The Quadratic Formula

The Quadratic Formula can be used to solve quadratic equations written in standard form:

$$ax^2 + bx + c = 0$$

$$\text{The Quadratic Formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To solve a Quadratic Equation using the QUADRATIC FORMULA:

Step 1: Make sure the quadratic equation is in standard form: $ax^2 + bx + c = 0$

Step 2: Identify the coefficients a , b , and c .

Step 4: Substitute these values into the Quadratic Formula

Step 5: Simplify your result completely.

Step 6: Verify result by graphing and finding the intersection point(s).

Do you wonder where this formula came from? Well, you can actually derive this formula directly from the quadratic equation in standard form $ax^2 + bx + c = 0$ using a factoring method called COMPLETING THE SQUARE. You will not be asked to use COMPLETING THE SQUARE in this class, but go through the information below and try to follow each step.

How to Derive the Quadratic Formula From $ax^2 + bx + c = 0$

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \quad [\text{Subtract } c \text{ from both sides then divide all by } a]$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2} \quad [\text{Take the coefficient of } x, \text{ divide it by } 2, \text{ square it, and add to both sides}]$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2} \quad [\text{Factor the left side. On the right side, get a common denominator of } 4a^2]$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}} \quad [\text{Combine the right side to one fraction then take square root of both sides}]$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a} \quad [\text{Simplify the square roots}]$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \quad [\text{Solve for } x]$$

$$x = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a} \quad [\text{Combine to obtain the final form for the Quadratic Formula}]$$

Problem 12	WORKED EXAMPLE– Solve Quadratic Equations Using the Quadratic Formula
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Solve the quadratic equation by using the Quadratic Formula. Verify your result by graphing and using the Intersection method.

Solve $3x^2 - 2 = -x$ using the quadratic formula.

Step 1: Write in standard form $3x^2 + x - 2 = 0$

Step 2: Identify $a = 3$, $b = 1$, and $c = -2$

Step 3: $x = \frac{-(1) \pm \sqrt{(1)^2 - 4(3)(-2)}}{2(3)} = \frac{-1 \pm \sqrt{1 - (-24)}}{6} = \frac{-1 \pm \sqrt{1 + 24}}{6} = \frac{-1 \pm \sqrt{25}}{6}$

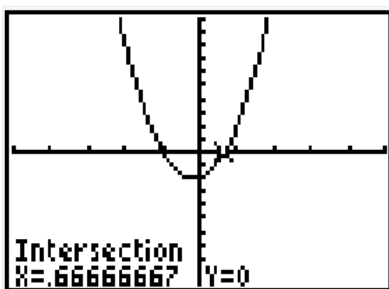
Step 4: Make computations for x_1 and x_2 as below and note the complete simplification process:

$$x_1 = \frac{-1 + \sqrt{25}}{6} = \frac{-1 + 5}{6} = \frac{4}{6} = \frac{2}{3} \qquad x_2 = \frac{-1 - \sqrt{25}}{6} = \frac{-1 - 5}{6} = \frac{-6}{6} = -1$$

Final solution $x = \frac{2}{3}$, $x = -1$

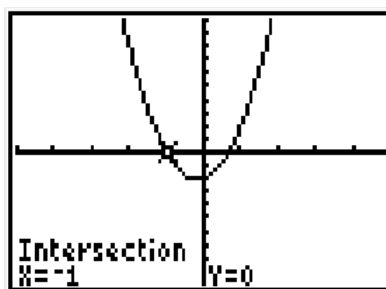
Step 5: Check by graphing.

Graphical verification of Solution $x = \frac{2}{3}$



[Note that $\frac{2}{3} \approx .6666667$]

Graphical verification of Solution $x = -1$



You can see by the graphs above that this equation is an example of the “Case 2” possibility of two, unique real number solutions for a given quadratic equation.

Problem 13 | **MEDIA EXAMPLE – Solve Quadratic Equations Using Quadratic Formula**

Solve each quadratic equation by using the Quadratic Formula. Verify your result by graphing and using the Intersection method.

$$\text{Quadratic Formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a) Solve $-x^2 + 3x + 10 = 0$

b) Solve $2x^2 - 4x = 3$

Problem 14 | **YOU TRY – Solve Quadratic Equations Using Quadratic Formula**

Solve $3x^2 = 7x + 20$ using the Quadratic Formula. Show all steps and simplify your answer. Verify your answer by graphing. Sketch the graph on a good viewing window (the vertex, vertical intercept, and any horizontal intercepts should appear on the screen). **Mark and label** the solutions on your graph.



Section 9.5 – Complex Numbers

Suppose we are asked to solve the quadratic equation $x^2 = -1$. Well, right away you should think that this looks a little weird. If I take any real number times itself, the result is always positive. Therefore, there is no REAL number x such that $x^2 = -1$. [Note: See explanation of Number Systems on the next page]

Hmmm...well, let's approach this using the Quadratic Formula and see what happens.

To solve $x^2 = -1$, need to write in standard form as $x^2 + 1 = 0$. Thus, $a = 1$ and $b = 0$ and $c = 1$.

Plugging these in to the quadratic formula, I get the following:

$$x = \frac{-0 \pm \sqrt{0^2 - 4(1)(1)}}{2(1)} = \frac{\pm \sqrt{-4}}{2} = \frac{\pm \sqrt{4(-1)}}{2} = \frac{\pm \sqrt{4}\sqrt{-1}}{2} = \frac{\pm 2\sqrt{-1}}{2} = \pm \sqrt{-1}$$

Well, again, the number $\sqrt{-1}$ does not live in the real number system nor does the number $-\sqrt{-1}$ yet these are the two solutions to our equation $x^2 + 1 = 0$.

The way mathematicians have handled this problem is to define a number system that is an extension of the real number system. This system is called the Complex Number System and it has, as its base defining characteristic, that equations such as $x^2 + 1 = 0$ can be solved in this system. To do so, a special definition is used and that is the definition that:

$$i = \sqrt{-1}$$

With this definition, then, the solutions to $x^2 + 1 = 0$ are just $x = i$ and $x = -i$ which is a lot simpler than the notation with negative under the radical.

When Will We See These Kinds of Solutions?

We will see solutions that involve the complex number “ i ” when we solve quadratic equations that never cross the x -axis. You will see several examples to follow that will help you get a feel for these kinds of problems.

Complex Numbers $a + bi$

Complex numbers are an extension of the real number system.
Standard form for a complex number is

$$a + bi$$

where a and b are real numbers,

$$i = \sqrt{-1}$$

Problem 15 | **WORKED EXAMPLE – Complex Numbers**

$$\begin{aligned} \text{a) } \sqrt{-9} &= \sqrt{9}\sqrt{-1} \\ &= 3\sqrt{-1} \\ &= 3i \end{aligned}$$

$$\begin{aligned} \text{b) } \sqrt{-7} &= \sqrt{7}\sqrt{-1} \\ &= \sqrt{7}i \text{ or } i\sqrt{7} \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{3 + \sqrt{-49}}{2} &= \frac{3 + \sqrt{49}\sqrt{-1}}{2} \\ &= \frac{3 + 7i}{2} \\ &= \frac{3}{2} + \frac{7}{2}i \end{aligned}$$

THE COMPLEX NUMBER SYSTEM

Complex Numbers:

All numbers of the form $a + bi$ where a, b are real numbers and $i = \sqrt{-1}$

Examples: $3 + 4i$, $2 + (-3)i$, $0 + 2i$, $3 + 0i$

Real Numbers – all the numbers on the REAL NUMBER LINE – include all RATIONAL NUMBERS and IRRATIONAL NUMBERS

Rational Numbers :

- ratios of integers
- decimals that terminate or repeat
- Examples:

$$0.50 = \frac{1}{2}, \quad -.75 = -\frac{3}{4},$$

$$0.43 = \frac{43}{100}, \quad 0.33 = \frac{33}{100}$$

Irrational Numbers

Examples: $\pi, e, \sqrt{5}, \sqrt{47}, \sqrt{11}$

- Decimal representations for these numbers never terminate and never repeat

Integers: Zero, Counting Numbers and their negatives
 $\{\dots -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots\}$

Whole Numbers: Counting Numbers and Zero
 $\{0, 1, 2, 3, 4, 5, 6, 7, \dots\}$

Counting Numbers
 $\{1, 2, 3, 4, 5, 6, 7, \dots\}$

Complex numbers are an extension of the real number system. As such, we can perform operations on complex numbers. This includes addition, subtraction, multiplication, and powers.

A complex number is written in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$

Extending this definition a bit, we can define $i^2 = (\sqrt{-1})^2 = \sqrt{-1} \cdot \sqrt{-1} = -1$

Problem 16	WORKED EXAMPLE – Operations with Complex Numbers
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Perform the indicated operations. Recall that $i^2 = -1$.

$$\begin{aligned} \text{a) } (8 - 5i) + (1 + i) &= 8 - 5i + 1 + i \\ &= 9 - 4i \end{aligned}$$

$$\begin{aligned} \text{b) } (3 - 2i) - (4 + i) &= 3 - 2i - 4 - i \\ &= -1 - 3i \end{aligned}$$

$$\begin{aligned} \text{c) } 5i(8 - 3i) &= 40i - 15i^2 \\ &= 40i - 15(-1) \text{ because } i^2 = -1 \\ &= 40i + 15 \\ &= 15 + 40i \end{aligned}$$

$$\begin{aligned} \text{d) } (2 + i)(4 - 2i) &= 8 - 4i + 4i - 2i^2 \\ &= 8 - 2i^2 \\ &= 8 - 2(-1) \\ &= 8 + 2 \\ &= 10 \\ &= 10 + 0i \end{aligned}$$

$$\begin{aligned} \text{e) } (3 - 5i)^2 &= (3 - 5i)(3 - 5i) \\ &= 9 - 15i - 15i + 25i^2 \text{ by FOIL} \\ &= 9 - 30i + 25i^2 \\ &= 9 - 30i + 25(-1) \text{ because } i^2 = -1 \\ &= 9 - 30i - 25 \\ &= -16 - 30i \end{aligned}$$

Problem 17	YOU TRY– Working with Complex Numbers
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Simplify each of the following and write your answers in the form $a + bi$.

$$\text{a) } \frac{15 - \sqrt{-9}}{3}$$

$$\text{b) } (10 + 4i)(8 - 5i)$$

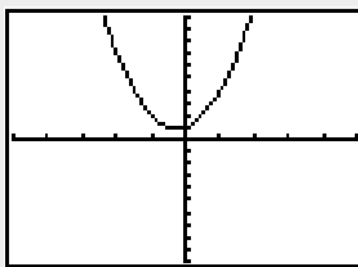
Section 9.6 – Complex Solutions to Quadratic Equations

Work through the following to see how to deal with equations that can only be solved in the Complex Number System.

Problem 18	WORKED EXAMPLE – Solving Quadratic Equations with Complex Solutions
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Solve $2x^2 + x + 1 = 0$ for x . Leave results in the form of a complex number, $a+bi$.

First, graph the two equations as Y1 and Y2 in your calculator and view the number of times the graph crosses the x-axis. The graph below shows that the graph of $y = 2x^2 + x + 1$ does not cross the x-axis at all. This is an example of our Case 3 possibility and will result in no Real Number solutions but two unique Complex Number Solutions.



To find the solutions, make sure the equation is in standard form (check).

Identify the coefficients $a = 2$, $b = 1$, $c = 1$.

Insert these into the quadratic formula and simplify as follows:

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(1)}}{2(2)} = \frac{-1 \pm \sqrt{1-8}}{4} = \frac{-1 \pm \sqrt{-7}}{4}$$

Break this into two solutions and use the $a+bi$ form to get

$$\begin{aligned} x_1 &= \frac{-1 + \sqrt{-7}}{4} \\ &= -\frac{1}{4} + \frac{\sqrt{-7}}{4} \\ &= -\frac{1}{4} + \frac{i\sqrt{7}}{4} \\ &= -\frac{1}{4} + \frac{\sqrt{7}}{4}i \end{aligned} \quad \text{and} \quad \begin{aligned} x_2 &= \frac{-1 - \sqrt{-7}}{4} \\ &= -\frac{1}{4} - \frac{\sqrt{-7}}{4} \\ &= -\frac{1}{4} - \frac{i\sqrt{7}}{4} \\ &= -\frac{1}{4} - \frac{\sqrt{7}}{4}i \end{aligned}$$

The final solutions are $x_1 = -\frac{1}{4} + \frac{\sqrt{7}}{4}i$, $x_2 = -\frac{1}{4} - \frac{\sqrt{7}}{4}i$

Remember that $\sqrt{-1} = i$ so $\sqrt{-7} = i\sqrt{7}$

Problem 19	MEDIA EXAMPLE – Solving Quadratic Equations with Complex Solutions
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Solve $x^2 + 4x + 8 = 1$ for x . Leave results in the form of a complex number, $a+bi$.

Problem 20	YOU TRY – Solving Quadratic Equations with Complex Solutions
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Solve $2x^2 - 3x = -5$ for x . Leave results in the form of a complex number, $a+bi$.

Work through the following problem to put the solution methods of graphing, factoring and quadratic formula together while working with the same equation.

Problem 21	YOU TRY – SOLVING QUADRATIC EQUATIONS
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Given the quadratic equation $x^2 + 3x - 7 = 3$, solve using the processes indicated below.

- a) Solve by graphing (use your calculator and the Intersection process). Sketch the graph on a good viewing window (the vertex, vertical intercept, and any horizontal intercepts should appear on the screen). **Mark and label** the solutions on your graph.



- b) Solve by factoring. Show all steps. Clearly identify your final solutions.

- c) Solve using the Quadratic Formula. Clearly identify your final solutions.