Lesson 6 – More Exponential Functions

Now that we have studied the basics of Exponential Functions, it is time to look at several specific concepts. In this lesson, we study Exponential Growth and Exponential Decay and look at ways to model and measure each. We also learn how to use our calculator to create an Exponential Model by using the Linear Regression tool.

Lesson Topics:

Section 6.1: Writing Exponential Models

- Characteristics of Exponential Functions
- Growth/Decay Rates
- Writing Exponential Growth/Decay models

Section 6.2: Doubling Time and Halving Time

- Writing Exponential Growth models – Doubling Time
- Writing Exponential Decay models – Halving Time

Section 6.3: Exponential Regression

Lesson 6 Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Name: _____ **Date:** _____

Mini-Lesson 6

Section 6.1 – Writing Exponential Models

Problem 1	**YOU TRY – Characteristics of Exponential Functions**

Given a function, $f(x) = ab^x$, respond to each of the following. Refer back to previous lessons as needed.

a) The variable x represents the _____ quantity.

b) $f(x)$ represents the _____ quantity.

c) The DOMAIN of $f(x)$ is _____

d) The RANGE of $f(x)$ is _____

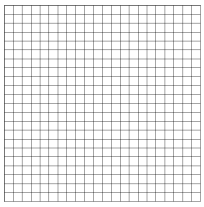
e) The INITIAL VALUE of $f(x)$ is _____

f) The VERTICAL INTERCEPT of $f(x)$ is (_____ , _____)

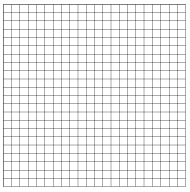
g) The HORIZONTAL INTERCEPT of $f(x)$ _____

h) The equation of the HORIZONTAL ASYMPTOTE of $f(x)$ is _____

i) On the graphing grid below, draw an exponential GROWTH function. In this case, what can you say about the GROWTH FACTOR b? $b >$ _____



a) On the graphing grid below, draw an exponential DECAY function. In this case, what can you say about the DECAY FACTOR b? _____ $< b <$ _____



Growth and Decay RATES

An exponential function $f(x) = ab^x$ grows (or decays) at a constant *percent rate, r.*

r = growth/decay rate in decimal form

GROWTH FACTOR: $b = 1 + r$ GROWTH RATE: $r = b - 1$

DECAY FACTOR: $b = 1 - r$ DECAY RATE: $r = 1 - b$

Problem 2 MEDIA EXAMPLE – Writing Exponential Growth/Decay Models

Complete the following table.

Exponential Function $y = ab^t$	Growth or Decay?	Initial Value a	Growth/Decay Factor b	Growth/Decay Rate, r (as a decimal)	Growth/Decay Rate, r (as a %)
$y = 812(0.71)^t$					
$y = 64.5(1.32)^t$					
	Growth	8.24			0.5%
	Decay	150			20%

Problem 3 | WORKED EXAMPLE – Writing Exponential Growth/Decay Models

Exponential Function $y = ab^t$	Growth or Decay?	Initial Value a	Growth/Decay Factor b	Growth/Decay Rate, r (as a decimal)	Growth/Decay Rate, r (as a %)
$y = 72(1.03)^t$	Growth	72	1.03	0.03	3%
$y = 44.1(0.92)^t$	Decay	44.1	0.92	0.08	8%
$y = (0.54)^t$	Decay	1	0.54	0.46	46%
$y = 2110(1.023)^t$	Growth	2110	1.023	0.023	2.3%
$y = 520(0.85)^t$	Decay	520	0.85	0.15	15%
$y = 3900(1.048)^t$	Growth	3900	1.048	0.048	4.8%

Problem 4 | YOU TRY – Writing Exponential Growth/Decay Models

Complete the following table.

Exponential Function $y = ab^t$	Growth or Decay?	Initial Value a	Growth/Decay Factor b	Growth/Decay Rate, r (as a decimal)	Growth/Decay Rate, r (as a %)
$y = 300(0.88)^t$					
$y = 213(1.2)^t$					
	Growth	177			9.8%
	Decay	5.41			7%

Section 6.2 – Doubling Time and Halving Time

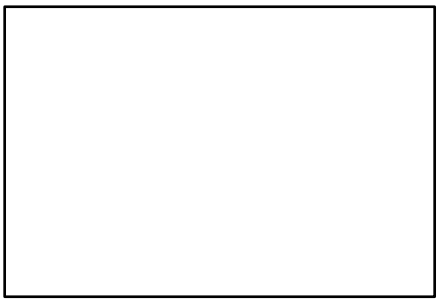
Problem 5	**MEDIA EXAMPLE – Writing Exponential Growth Models / Doubling Time**

In 2001, the population of a particular city was 22,395 with an identified growth rate of 6.2% per year. Assume that this growth rate is fairly consistent from year to year.

a) Write the EXPONENTIAL GROWTH MODEL for this situation.

b) What is the approximate population of the city in 2006? Be sure and round to the nearest person.

c) Estimate the number of years (to the nearest whole year) that it will take for the population to DOUBLE. In what actual year will this take place?



Problem 6	**WORKED EXAMPLE - Writing Exponential Growth Models/Doubling Time**

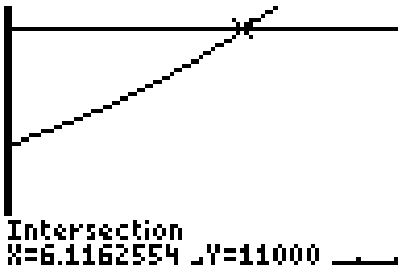
A city has a current population of 5500 people with a growth rate of 12% per year. Write the exponential model for this population and determine the time (to the nearest year) for the population to double.

First, determine the EXPONENTIAL MODEL using the information given in the problem.

- Given: Initial population = 5500
- Given: Growth rate of 12% per year
- Formula to use: $P(t) = ab^t$
- a = 5500 (initial population)
- To find b, convert 12% to a decimal (.12), Then, since the population grows, $b = 1 + .12 = 1.12$ (This value is also called the GROWTH FACTOR).
- Write the model: $P(t) = 5500(1.12)^t$

Second, determine the time for the population to double (DOUBLING TIME)

- Given: $P(t) = 5500(1.12)^t$, initial population = 5500
- Goal: Determine the time for the population to double. Another way to say this is, "find the value of t when the population is twice the initial population" (i.e. find t when $P(t) = 2(5500) = 11000$).
- Mathematically, we want to solve the equation: $5500(1.12)^t=11000$
- Use calculator entering Y1 = $5500(1.12)^t$ and Y2 = 11000. Use window X[0..10], Y[0..12000] then 2^{nd}>Calc>5:Intersect to get t = 6.12. (See graph below). Round to get t = 6.



Result: The population will double in about 6 years.

Steps to Write an Exponential Growth Model Given the Rate of Growth

- Determine initial value of the model (i.e. initial population, initial investment, initial salary, etc.). This is the value of the model at time t = 0 and the number will be your number for "a".
- Write the given rate as a decimal and ADD it to 1. This is your value for "b" (GROWTH FACTOR).
- Write the model using appropriate function notation (i.e. $P(t) = ab^t$, $V(t) = ab^t$, $S(t) = ab^t$, etc.)

Steps to Determine Doubling Time

- Start with an exponential growth model, i.e. $P(t) = ab^t$
- Set up the equation $ab^t = 2a$
- Solve by graphing and INTERSECTION method

Problem 7	**YOU TRY – Writing Exponential Growth Models / Doubling Time**

After graduating from college in 2010, Sara accepts a job that pays $52,000 per year. At the end of each year, she expects to receive a 3% raise.

a) Let t represent the number of years Sara works at her new job. Write the exponential growth function, $S(t)$, that models her annual salary given the information above.

Initial Salary (a value): _____

Given growth rate as a decimal: _____

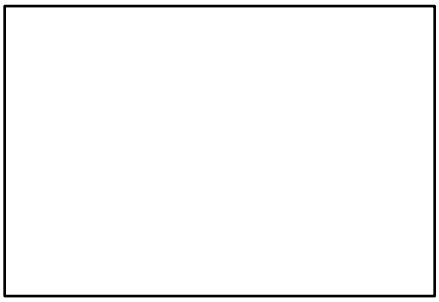
Growth factor (b value): _____

Write the model: $S(t) = \text{InitialValue}(\text{GrowthFactor})^t =$ _____

b) If Sara's salary continues to increase at the rate of 3% each year, determine how much will she will make in 2015. Show your work clearly here.

c) How many years will she have to work before her salary will be double what it was in 2010 (assuming the same growth rate)? Be sure to set up and clearly identify the DOUBLING equation. Then, draw a sketch of the graph you obtain when using the INTERSECTION method to solve. Round to the nearest WHOLE year.

DOUBLING EQUATION: _____



DOUBLING TIME (Rounded to nearest whole year): _____

Problem 8	**MEDIA EXAMPLE – Writing Exponential Decay Models / Halving Time**

The 2000 U.S. Census reported the population of Tulsa, Oklahoma to be 382,872. Since the 2000 Census, Tulsa's population has been decreasing at approximately 2.6% per year.

a) Write an EXPONENTIAL DECAY MODEL, $P(t)$, that predicts the population of Tulsa, OK at any time t.

b) Use the function you wrote for $P(t)$ to predict the population of Tulsa, OK in 2013.

c) In how many years will the population of Tulsa decrease to 300,000 people (round to the nearest whole year)?

d) In how many years will the population of Tulsa decrease to HALF of the initial (2000) population? Round to the nearest whole year.

Problem 9	**WORKED EXAMPLE – Writing Exponential Decay Models / Halving Time**

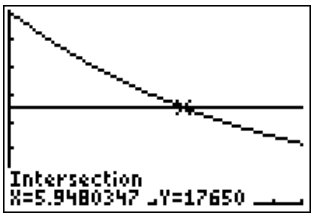
In 2012, Shannon purchased a new Ford Mustang GT convertible for $35,300. Since then, the value of the car has decreased at a rate of 11% each year.

First, determine the EXPONENTIAL MODEL using the information given in the problem.

- Given: Purchase price = $35,300
- Given: Decay rate of 11% per year
- Formula to use: $V(t) = ab^t$
- a = 35,300 (initial value)
- To find b, convert 11% to a decimal (0.11), Since the population decays, $b = 1 - .11 = 0.89$ (This value is also called the DECAY FACTOR).
- Write the model: $V(t) = 35300(0.89)^t$

Second, determine the time for the price to halve (HALF-LIFE or HALVING TIME)

- Given: $V(t) = 35300(0.89)^t$, initial price = $35,300
- Goal: Determine the time for the value of the car to halve. Another way to say this is, "find the value of t when the value is half the initial purchase price" (i.e. find t when $V(t) = 0.5(35,300) = 17,650$).
- Mathematically, we want to solve the equation: $35300(0.89)^t = 17650$
- Use your calculator and enter $Y1 = 35300(0.89)^t$ and $Y2 = 17650$. Use window X[0..10], Y[0..35300] then 2nd>Calc>5:Intersect to get $t = 5.95$ (See graph below).



- Result: The value of the car will be worth half the initial purchase price in about 6 years.

Steps to Write an Exponential Decay Model Given the Rate of Decay

- Determine initial value of the model (i.e. initial population, initial investment, initial salary, etc.). This is the value of the model at time $t = 0$ and the number will be your number for "a".
- Write the given rate as a decimal and SUBTRACT it from 1. This is "b"(DECAY FACTOR).
- Write the model using appropriate function notation (i.e. $P(t) = ab^t$, $V(t) = ab^t$, $S(t) = ab^t$, etc.)

Steps to Determine Halving Time (also called Half-Life)

- Start with an exponential growth model, i.e. $P(t) = ab^t$
- Set up the equation $ab^t = 0.5a$
- Solve by graphing and INTERSECTION method

Problem 10	**YOU TRY – Writing Exponential Decay Models / Halving Time**

In 1970, the population of Buffalo, New York had a population of 462,768 people. Assume the population decreased by 1.4% each year from 1970 to 2000.

a) Let t represent the number of years since 1970 (i.e. your starting year is 1970 so t=0 in this year). Write the exponential decay function, $P(t)$, that models the annual population given the information above.

Initial Population (a value): _____ Given DECAY RATE as a decimal: _____

Take 1 – the DECAY RATE decimal to identify your DECAY FACTOR (b value):

_____

Write the model: $P(t) = InitialValue(DecayFactor)^t =$ _____

b) If the population continues to decrease at the rate above, determine how many people lived in Buffalo in 1989. Show your work clearly here.

c) How many years will it take for the Buffalo population to decrease to half what it was in 1970 (assuming the same decay rate)? Be sure to set up and clearly identify the HALVING equation. Then, draw a sketch of the graph you obtain when using the INTERSECTION method to solve. Round to the nearest WHOLE year.

HALVING EQUATION: _____

HALVING TIME (Rounded to nearest whole year): _____

Section 6.3 – Exponential Regression

As with LINEAR FUNCTIONS, we can work with a data table and, if appropriate, model that data using EXPONENTIAL REGRESSION. The steps are almost the same as those followed for LINEAR REGRESSION.

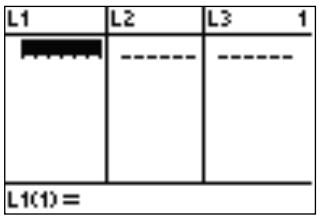
Problem 11	**WORKED EXAMPLE–Exponential Regression**

The table below shows the population, P, in a given state after t years.

t (years)	Population
5	5,234,456
10	4,892,345
15	4,012,345

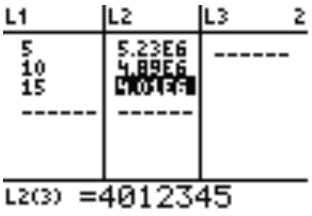
Use the Exponential Regression feature of your calculator to generate a mathematical model for this situation. Round "a" to the nearest whole number and "b" to 3 decimals.

- Press STAT>EDIT>ENTER to show data entry area. The STAT button is on the second row, third column.

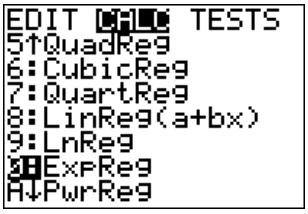


Data entry area should be blank to begin. To clear, go column by column. Scroll to column header using the arrow keys then press Clear>Enter. Use the arrow keys to move back and forth.

[Note: The numbers for L2 are displayed using scientific notation (the E notation) since they are too long for the column width. Scroll to each number and see its full representation in the bottom of the screen. See example highlighted at right.

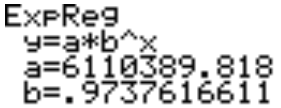


- Press STAT>CALC>0:ExpReg>ENTER>ENTER



Thus, your exponential function (with values rounded as the problem indicates) is $y = 6110390(0.974)^x$. Convert this to function notation with the appropriate variales to get

$$P(t) = 6110390(0.974)^t.$$



Problem 12	**YOU TRY – Exponential Regression**

Determine the exponential regression equation that models the data below:

t	0	1	2	4	6	9	12	16
$P(t)$	125	75	50	32	22	16	10	5.7

When you write your final equation, round "a" to 1 decimal place and "b" to three decimal places.

a) Write exponential regression equation in the form $y = ab^x$: _____

Rewrite exponential regression equation in the form $P(t) = ab^t$: _____

b) Use your graphing calculator to generate a scatterplot of the data *and* the graph of the regression equation on the same screen. You must use an appropriate viewing window. In the space below, draw what you see on your calculator screen, and write down the viewing window you used.

Xmin=_____

Xmax=_____

Ymin=_____

Ymax=_____

c) What is the rate of decay (as a %) for this function? _____

d) Determine $P(20)$. Show your work, and write the corresponding ordered pair result. Round to two decimal places.

e) Using your equation from part a, determine t when $P(t) = 28$. Show your work. Write the corresponding ordered pair result. Round to two decimal places.

Problem 13	**YOU TRY – Exponential Regression**

The table below shows the value, V, of an investment (in thousands of dollars) after n years.

n	0	3	5	10	15	22
V(n)	4.63	5.92	6.88	10.23	15.21	26.39

a) Use your calculator to determine the exponential regression equation that models the set of data above. Round the "a" value to two decimals, and round the "b" value to three decimals. Use the indicated variables and proper function notation.

b) Based on the equation found in part a), at what percent rate is the value of this investment increasing each year?

c) Determine V(12), and write your answer in a complete sentence. Round your answer to two decimal places.

d) How long will it take for the value of this investment to reach \$100,000? Round your answer to two decimal places. Write your answer in a complete sentence.

e) How long will it take for the value of the investment to double? Round your answer to two decimal places. Write your answer in a complete sentence.