Lesson 2 – Functions and Function Operations

As we continue to work with more complex functions it is important that we are comfortable with Function Notation, operations on Functions and operations involving more than one function. In this lesson, we study using proper Function Notation and then spend time learning how add, subtract, multiply and divide Functions, both algebraically and when the functions are represented with a tables or graphs. Finally, we take a look at a couple of real world examples that involve operations on functions.

Lesson Topics:

Lesson 2 Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Name: ______________________________ **Date:** ________________

Mini-Lesson 2

Section 2.1 – Combining Functions

Function notation can be expanded to include notation for the different ways we can combine functions as described below.

Basic Mathematical Operations
The basic mathematical operations are: addition, subtraction, multiplication, and division. When working with function notation, these operations will look like this:

Addition	Subtraction	Multiplication	Division
$f(x)+g(x)$	$f(x)-g(x)$	$f(x)\cdot g(x)$	$\frac{f(x)}{g(x)}$ $g(x)\neq 0$

Many of the problems we will work in this lesson are problems you may already know how to do. You will just need to get used to some new notation.

We will start with the operations of addition and subtraction**.**

Problem 1	**WORKED EXAMPLE – Adding and Subtracting Functions**

Given $f(x) = 2x^2 + 3x - 5$ and $g(x) = -x^2 + 5x + 1$.

a) Find $f(x) + g(x)$

$$f\ (x) + g(x) = (2x^2 + 3x - 5) + (-x^2 + 5x + 1)$$

b) Find $f(x) - g(x)$

$$f\ (x) - g(x) = (2x^2 + 3x - 5) - (-x^2 + 5x + 1)$$

c) Find $f(1) - g(1)$

$$f\ (1) - g(1) = [2(1)^2 + 3(1) - 5] - [-(1)^2 + 5(1) + 1]$$

Problem 2	MEDIA EXAMPLE – Adding and Subtracting Functions

Given $f(x) = 3x^2 + 2x - 1$ and $g(x) = x^2 + 2x + 5$:

a) Find $f(x) + g(x)$

b) Find $f(x) - g(x)$

Problem 3	YOU TRY – Adding and Subtracting Functions

Given $f(x) = x^2 + 4$ and $g(x) = x^2 + 1$, determine each of the following. Show complete work.

a) Find $f(2) + g(2)$

b) Find $f(x) - g(x)$

c) Find $f(2) - g(2)$

Function Multiplication and the Multiplication Property of Exponents

When multiplying functions, you will often need to work with exponents.
The following should be familiar to you and will come into play in the examples below:

MULTIPLICATION PROPERTY OF EXPONENTS
Let *m* and *n* be rational numbers.
To multiply powers of the same base, keep the base and add the exponents:

$$a^m \cdot a^n = a^{m+n}$$

Problem 4	WORKED EXAMPLE – Function Multiplication

a) Given $f(x) = -8x^4$ and $g(x) = 5x^3$, find $f(x) \cdot g(x)$

$f(x) \cdot g(x) = (-8x^4)(5x^3)$

Reorder using Commutative Property
Simplify using the Multiplication Property of Exponents
Final Result

b) Given $f(x) = -3x$ and $g(x) = 4x^2 - x + 8$, find $f(x) \cdot g(x)$

$f(x) \cdot g(x) = (-3x)(4x^2 - x + 8)$ Apply the Distributive Property
$f(x) \cdot g(x) = -12x^3 + 3x^2 - 24x$ Remember the rules of exp.
$(-3x)(4x^2) = (-3)(4)(x^1)(x^2)$
Final Result

c) Given $f(x) = 3x + 2$ and $g(x) = 2x - 5$, find $f(x) \cdot g(x)$

$f(x) \cdot g(x) = (3x + 2)(2x - 5)$ Use FOIL
Remember the rules of exp.
$(3x)(2x) = (3)(2)(x)(x)$
Combine Like Terms
Final Result

Problem 5	MEDIA EXAMPLE – Function Multiplication

Given $f(x) = 3x + 2$ and $g(x) = 2x^2 + 3x + 1$, find $f(x) \cdot g(x)$

Problem 6	YOU TRY – Function Multiplication

For each of the following, find $f(x) \cdot g(x)$

a) $f(x) = 3x - 2$ and $g(x) = 3x + 2$

b) $f(x) = 2x^2$ and $g(x) = x^3 - 4x + 5$

c) $f(x) = 4x^3$ and $g(x) = -6x$

Function Division and the Division Property of Exponents

When dividing functions, you will also need to work with exponents of different powers. The following should be familiar to you and will come into play in the examples below:

DIVISION PROPERTY OF EXPONENTS

Let m, n be rational numbers. To divide powers of the same base, keep the base and subtract the exponents.

$$\frac{a^m}{a^n} = a^{m-n} \text{ where } a \neq 0$$

Problem 7	WORKED EXAMPLE – Function Division

For each of the following, find $\frac{f(x)}{g(x)}$. Use only positive exponents in your final answer.

a) $f(x) = 15x^{15}$ and $g(x) = 3x^9$

$$\frac{f(x)}{g(x)} = \frac{15x^{15}}{3x^9}$$
$$= 5x^{15-9}$$
$$= 5x^6$$

NEGATIVE EXPONENTS

If $a \neq 0$ and n is a rational number, then $a^{-n} = \frac{1}{a^n}$

b) $f(x) = -4x^5$ and $g(x) = 2x^8$

$$\frac{f(x)}{g(x)} = \frac{-4x^5}{2x^8}$$
$$= -2x^{5-8}$$
$$= -2x^{-3}$$
$$= -\frac{2}{x^3}$$

Problem 8	**MEDIA EXAMPLE – Function Division**

For each of the following, determine $\frac{f(x)}{g(x)}$. Use only positive exponents in your final answer.

a) $f(x) = 10x^4 + 3x^2$ and $g(x) = 2x^2$

b) $f(x) = -12x^5 + 8x^2 + 5$ and $g(x) = 4x^2$

Problem 9	**YOU TRY – Function Division**

For each of the following, determine $\frac{f(x)}{g(x)}$. Use only positive exponents in your final answer.

a) $f(x) = 25x^5 - 4x^7$ and $g(x) = -5x^4$

b) $f(x) = 20x^6 - 16x^3 + 8$ and $g(x) = -4x^3$

Functions can be presented in multiple ways including: equations, data sets, graphs, and applications. If you understand function notation, then the process for working with functions is the same no matter how the information if presented.

Problem 10	**MEDIA EXAMPLE – Working with Functions in Table Form**

Functions $f(x)$ and $g(x)$are defined in the tables below. Find a – e below using the tables.

x	–3	–2	0	1	4	5	8	10	12
$f(x)$	8	6	3	2	5	8	11	15	20

x	0	2	3	4	5	8	9	11	15
$g(x)$	1	3	5	10	4	2	0	–2	–5

a) $f(1) =$

b) $g(9) =$

c) $f(0) + g(0) =$

d) $g(5) - f(8) =$

e) $f(0) \cdot g(3) =$

Problem 11	**YOU TRY – Working with Functions in Table Form**

Given the following two tables, complete the third table. Show work in the table cell for each column. The first one is done for you.

x	0	1	2	3	4
$f(x)$	4	3	–2	0	1

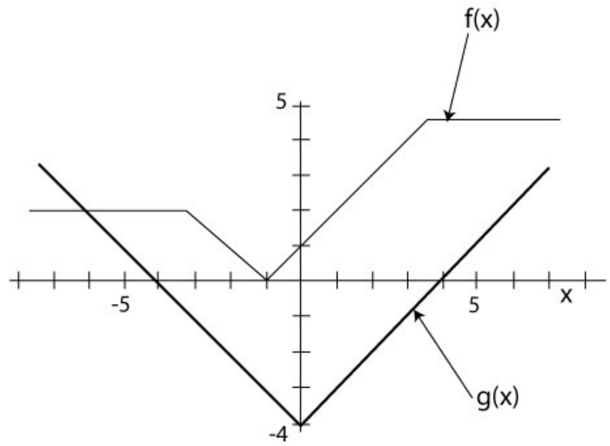
x	0	1	2	3	4
$g(x)$	6	–3	4	–2	2

x	0	1	2	3	4
$f(x)+g(x)$	$f(0)+g(0)$	$f(1)+g(1)$			

If you remember that graphs are just infinite sets of ordered pairs and if you do a little work ahead of time (as in the example below) then the graphing problems are a lot easier to work with.

Problem 12	YOU TRY – Working with Functions in Graph Form

Use the graph to determine each of the following. Assume integer answers. The graph of g is the graph in bold.



Complete the following ordered pairs from the graphs above. Use the information to help you with the problems below. The first ordered pair for each function has been completed for you.

f: (–7, 2), (–6,), (–5,), (–4,), (–3,), (–2,), (–1,), (0,), (1,), (2,), (3,), (4,), (5,), (6,), (7,)

g: (–7, 3), (–6,), (–5,), (–4,), (–3,), (–2,), (–1,), (0,), (1,), (2,), (3,), (4,), (5,), (6,), (7,)

a) $g(4)$ = ________________________

b) $f(2)$ = ____________________

c) $g(0)$ = ________________________

d) $f(-6)$ = ____________________

e) If $f(x) = 0$, x = ____________________

f) If $g(x) = 0$, x = ____________________

g) If $f(x) = 1$, x = ____________________

h) If $g(x) = -4$, x = ____________________

i) $f(-1) + g(-1)$ = ____________________

j) $g(-6) - f(-6)$ = ____________________

k) $f(1) * g(-2)$ = ____________________

l) $\dfrac{g(6)}{f(-1)}$ = ____________________

Section 2.2 – Applications of Function Operations

One of the classic applications of function operations is the forming of the Profit function, P(x) by subtracting the cost function, C(x), from the revenue function, R(x) as shown below.

Profit = Revenue – Cost

Given functions $P(x)$ = Profit, $R(x)$ = Revenue, and $C(x)$ = Cost:

$$P(x) = R(x) - C(x)$$

Problem 13	**MEDIA EXAMPLE – Cost, Revenue, Profit**

A local courier service estimates its monthly operating costs to be $1500 plus $0.85 per delivery. The service generates revenue of $6 for each delivery. Let x = the number of deliveries in a given month.

a) Write a function, $C(x)$, to represent the monthly costs for making x deliveries per month.

b) Write a function, $R(x)$, to represent the revenue for making x deliveries per month.

c) Write a function, $P(x)$, that represents the monthly profits for making x deliveries per month.

d) Determine algebraically the break-even point for the function $P(x)$ and how many deliveries you must make each month to begin making money. Show complete work. Write your final answer as a complete sentence.

e) Determine the break-even point graphically by solving the equation $P(x) = 0$. Explain your work and show the graph with appropriate labels. Write your final answer as a complete sentence.

Problem 14	**YOU TRY – Cost, Revenue, Profit**

February is a busy time at Charlie's Chocolate Shoppe! During the week before Valentine's Day, Charlie advertises that his chocolates will be selling for \$1.80 a piece (instead of the usual \$2.00 each). The fixed costs to run the Chocolate Shoppe total \$450 for the week, and he estimates that each chocolate costs about \$0.60 to produce. Charlie estimates that he can produce up to 3,000 chocolates in one week.

a) Write a function, $C(n)$, to represent Charlie's total costs for the week if he makes n chocolates.

b) Write a function, $R(n)$, to represent the revenue from the sale of n chocolates during the week before Valentine's Day.

c) Write a function, $P(n)$, that represents Charlie's profit from selling n chocolates during the week before Valentine's Day. Show complete work to find the function.

d) Interpret the meaning of the statement $P(300) = -90$.

e) Determine the Practical Domain and Practical Range for $P(n)$, then use that information to define an appropriate viewing window for the graph of $P(n)$. Sketch the graph from your calculator in the space provided.

Practical Domain:

Practical Range:

f) How many chocolates must Charlie sell in order to break even? Show complete work. Write your final answer as a complete sentence. **Mark the break even point on the graph above.**

Section 2.3 – Composition of Functions

Composition of Functions

Function Composition is the process by which the OUTPUT of one function is used as the INPUT for another function. Two functions $f(x)$ and $g(x)$ can be composed as follows:

$f(g(x))$, where the function $g(x)$ is used as the INPUT for the function $f(x)$.

OR

$g(f(x))$, where the function $f(x)$ is used as the INPUT for the function $g(x)$.

Problem 15	WORKED EXAMPLE – Composition of Functions

Let $f(x) = 5 - x$ and $g(x) = 3x + 4$. Evaluate each $f(g(x))$, $g(f(x))$, and $f(g(7))$.

$$\begin{aligned} f(g(x)) &= f(3x+4) \\ &= 5-(3x+4) \\ &= 5-3x-4 \\ &= -3x+1 \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(5-x) \\ &= 3(5-x)+4 \\ &= 15-3x+4 \\ &= -3x+19 \end{aligned}$$

To evaluate $f(g(7))$, always start with the "inside" function. In this case, $g(7)$.

$$\begin{aligned} g(7) &= 3(7)+4 \\ &= 21+4 \\ &= 25 \end{aligned}$$

Then plug this result (output) into $f(x)$.

$$\begin{aligned} f(g(7)) &= f(25) \\ &= 5-(25) \\ &= -20 \end{aligned}$$

Problem 16	**MEDIA EXAMPLE – Composition of Functions**

Let $A(x) = 2x + 1$ and $B(x) = 3x - 5$. Evaluate each of the following.

$A(B(x)) =$ $B(A(x)) =$

$A(B(4)) =$ $B(A(4)) =$

Problem 17	**YOU TRY– Composition of Functions**

Let $f(x) = 4 - 3x$ and $g(x) = x - 8$. Evaluate each of the following.

a) $f(g(x)) =$ b) $g(f(5)) =$

Problem 18	**MEDIA EXAMPLE – Composition of Functions Given in Table Form**

The functions $f(x)$ and $g(x)$ are defined by the tables below.

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	4	11	10	8	6	5	8	2	6	9

x	1	2	3	4	5	6	7	8	9	10
$g(x)$	3	8	4	10	2	5	11	0	4	1

$f(g(5)) =$

$f(g(1)) =$

$g(f(4)) =$

$f(f(1)) =$

Problem 19	**YOU TRY – Composition of Functions Given in Table Form**

The functions A(x) and B(x) are defined by the tables below.

x	A(x)
0	5
1	7
2	3
3	6
4	1
5	2
6	11
7	3

x	B(x)
0	7
1	0
2	4
3	2
4	6
5	1
6	3
7	15

a) A(B(4)) = ____________________________

b) B(A(1)) = ____________________________

c) B(A(7)) = ____________________________

d) A(B(1)) = ____________________________

Section 2.4 – Applications of Function Composition

Problem 20	**MEDIA EXAMPLE – Applications of Function Composition**

Lisa makes \$18 per hour at her new part-time job.

a) Write a function, I, to represent Lisa's income for the week if she works h hours. Complete the table below.

$I(h)$ = ________________________________

h	5	10	15	20
$I(h)$				

b) Lisa puts 10% of her salary in her bank savings every week and \$10 into her piggy bank for a rainy day. Write a function, S, to represent the total amount of money she saves each week if her income is I dollars. Complete the table below.

$S(I)$ = ________________________________

I	90	180	270	360
$S(I)$				

c) Using the information above, write a formula for $S(I(h))$ and complete the table below.

$S(I(h))$ = ________________________________

h	5	10	15	20
$S(I(h))$				

d) What information does the function $S(I(h))$ provide in this situation? Be sure to identify the input and output quantities.

e) Interpret the meaning of the statement $S(I(10)) = 28$. Include all appropriate units.

Problem 21	**YOU TRY – Applications of Function Composition**

A resort hotel in Scottsdale, AZ charges \$1800 to rent a reception hall, plus \$58 per person for dinner and open bar. The reception hall can accommodate up to 200 people.

a) Write a function, T, to represent the total cost to rent the reception hall if n people attend the reception.

$T(n)$ = ________________________

b) During the summer months, the hotel offers a discount of 15% off the total bill, T. Write a function, D, to represent the discounted cost if the total bill was \$$T$.

$D(T)$= ________________________

c) Using the information above, write a formula for $D(T(n))$ and complete the table below.

$D(T(n))$ = __________________________________

n	0	50	100	150	200
$D(T(n))$					

d) What information does the function $D(T(n))$ provide in this situation? Be sure to identify the input and output quantities.

e) Interpret the meaning of the statement $D(T(100)) = 6460$. Include all appropriate units.

f) Determine the maximum number of people that can attend the reception for \$5,000 (after the discount is applied)?