Lesson 12 Practice Problems

Functions

1. In the space below, draw a graph that represents a function, and a graph that does NOT represent a function.

   ![Function Graph](image1)
   ![Not a Function Graph](image2)

2. Are these functions? Circle yes or no.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Input</th>
<th>Output</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>-23</td>
<td>695</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td>85</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>2</td>
<td>5</td>
<td>302</td>
<td>-80</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>2</td>
<td>6</td>
<td>12</td>
<td>0</td>
</tr>
</tbody>
</table>

   Yes   No   Yes   No   Yes   No

3. Are these functions? Circle yes or no.

   a) \{(-2, -4), (6, -4), (0, 0), (5, 0)\} Yes No
   
   b) \{(1, 1), (2, 2), (3, 3), (4, 4)\} Yes No
   
   c) \{(1, -8), (5, 2), (1, 6), (7, -3)\} Yes No

4. Are these functions? Circle yes or no.

   ![Yes Graph](image3)
   ![Yes Graph](image4)
   ![Yes Graph](image5)

   Yes   No   Yes   No   Yes   No
5. Answer true or false:
   a) The sales tax is a function of the price of an item.
   b) The numerical grade in this course is a function of the letter grade.
   c) Cooking time for a turkey is a function of the weight of the bird.
   d) The letter grade on a true/false quiz is a function of the number of questions answered correctly.

6. The function $r(x)$ is defined by the following table of values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>9</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r(x)$</td>
<td>-9</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

   a) $r(9) =$ ___________
   b) $r(3) =$ ___________
   c) $r(________) = 1$
   d) $r(________) = 3$
   e) The domain of $r(x)$ is { ___________________________________________ }
   f) The range of $r(x)$ is { ____________________________________________ }

7. Consider the function $g = \{ (2, 5), (0, 6), (5, 8), (-3, 7) \}$

   a) $g(0) =$ ___________
   b) $g(5) =$ ___________
   c) $g(________) = 7$
   d) $g(________) = 5$
   e) The domain of $g$ is { ___________________________________________ }
   f) The range of $g$ is { ____________________________________________ }
8. Given $f(4) = 8$, $f(3) = 11$, $f(0) = 6$

a) The domain of $f$ is {_________________________________________}

b) The range of $f$ is {_________________________________________}

c) Write the function $f$ as a set of ordered pairs.

9. The graph of $f(x)$ is given below.

a) Domain: _______________________

b) Range _______________________

c) $f(-3) =$_________

d) $f(0) =$_________

e) $f(x) = 4$ when $x =$_________

f) $f(x) = 0$ when $x =$_________

10. The graph of $f(x)$ is given below.

a) Domain: _______________________

b) Range _______________________

c) $f(3) =$_________

d) $f(0) =$_________

e) $f(x) = -2$ when $x =$_________

f) $f(x) = 0$ when $x =$_________
11. The graph of \( f(x) \) is given below.

- a) Domain: _______________________
- b) Range: _______________________
- c) \( f(-1) = _________ \)
- d) \( f(0) = _________ \)
- e) \( f(x) = -5 \) when \( x = _________ \)

12. Let \( W(p) = p^2 - 9p + 20 \). Show all steps. Write each answer in function notation and as an ordered pair.

- a) Determine \( W(-10) \).
- b) For what value(s) of \( p \) is \( W(p) = 0? \)

13. Let \( h(x) = 4 \). Show all steps. Write each answer in function notation and as an ordered pair.

- a) Determine \( h(5) \).
- b) Determine \( h(81) \).
14. Let \( p(x) = \frac{40}{2x} \). Show all steps. Write each answer in function notation and as an ordered pair.
   a) Determine \( p(5) \).
   b) For what value of \( x \) is \( p(x) = \frac{1}{4} \)?

   c) Determine the domain of \( p(x) \).

15. The functions \( A \) and \( B \) are defined by the following tables

   \[
   \begin{array}{c|cccccccccc}
   x & 0 & 2 & 3 & 4 & 5 & 8 & 9 & 11 & 15 & 12 \\
   \hline
   A(x) & 8 & 6 & 3 & 2 & 5 & 8 & 11 & 15 & 20 \\
   \end{array}
   \]

   \[
   \begin{array}{c|cccccccccc}
   x & -3 & -2 & 0 & 1 & 4 & 5 & 8 & 10 & 12 \\
   \hline
   B(x) & 1 & 3 & 5 & 10 & 4 & 2 & 0 & -2 & -5 \\
   \end{array}
   \]

   Determine the values for each of the following.
   a) \( B(3) = \) _________
   b) \( A(8) = \) _________
   c) \( A(0) + B(0) = \) _________
   d) \( A(8) - B(8) = \) _________
   e) \( A(4) \cdot B(4) = \) _________
   f) \( \frac{A(5)}{B(5)} = \) _________
   g) \( A(B(0)) = \) _________
   h) \( B(A(10)) = \) _________
   i) \( B(B(3)) = \) _________
16. Let \( p(x) = x^2 + 2x + 3 \) and \( r(x) = x - 5 \). Determine each of the following. Show all work. Box your answers.

   a) \( p(x) - r(x) = \)

   b) \( p(0) \cdot r(0) = \)

   c) \( p(-2) + r(-2) = \)

   d) \( r(7) - p(7) = \)

   e) \( p(r(x)) = \)

   f) \( r(p(7)) = \)
Linear Functions

17. Darby signs a 48-month lease agreement for a new Chevrolet Camaro 2LT convertible. The function $T(n) = 3491.88 + 580.85n$ gives the total amount paid $n$ months after signing.

a) Using complete sentences, interpret $T(12) = 10462.08$ in the context of the story.

b) Determine $T(24)$ and write a sentence explaining the meaning of your answer in this situation.

c) Determine the value of $n$ if $T(n) = 30,000$. Write a sentence explaining the meaning of your answer in this situation.

d) Identify the slope of $T(n)$ and interpret its meaning in a complete sentence.

e) Identify the vertical intercept of $T(n)$. Write it as an ordered pair and interpret its meaning in a complete sentence.

f) Determine the practical domain of $T(n)$. Use inequality notation. Include units.

g) Determine the practical range of $T(n)$. Use inequality notation. Include units.
18. A candy company has a machine that produces candy canes. The table below is a partial list of the relationship between the number of minutes the machine is operating and the number of candy canes produced by the machine during that time period.

<table>
<thead>
<tr>
<th>Minutes $t$</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Candy Canes $C(t)$</td>
<td>12</td>
<td>20</td>
<td>32</td>
<td>48</td>
<td>60</td>
</tr>
</tbody>
</table>

a) Include units. $C(12) =$ ______________________________

b) In a complete sentence and including all appropriate units, explain the meaning of your answer in part a).

c) Determine the average rate of change of $C(t)$ from 5 minutes to 8 minutes. Interpret your answer in a complete sentence.

d) Is $C(t)$ a linear function? If yes, identify the slope and write the equation in $C(t) = mt + b$ form.

e) How many candy canes will this machine produce in one hour?
19. The following table shows the distance of rocket from Earth in 100,000’s of miles as it travels towards Mars.

<table>
<thead>
<tr>
<th>Number of Days</th>
<th>1</th>
<th>3</th>
<th>7</th>
<th>12</th>
<th>18</th>
<th>27</th>
<th>37</th>
<th>44</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from Earth (100,000’s of miles)</td>
<td>5.1</td>
<td>15.9</td>
<td>35.2</td>
<td>61.1</td>
<td>89.7</td>
<td>137.2</td>
<td>183.5</td>
<td>223.0</td>
</tr>
</tbody>
</table>

a) Let $t = \text{the number of days since the rocket was launched}$ and $D(t) = \text{the distance from Earth}$. Use your calculator to determine the linear regression equation for the data (round all values to two decimal points).

b) Use your regression equation to estimate the rocket’s distance from Earth 23 days after launch.

c) Use your regression equation to estimate when the rocket will reach Mars if Mars’ distance from Earth is approximately 127 million miles.

d) Determine $D(20)$ and write a sentence explaining the meaning of your answer.

e) Use your regression equation to estimate the velocity of the rocket. Make sure to include units in your answer.
20. Bill’s car breaks down and he calls a tow company. The company’s charges can be found by using the linear function, $T(x) = 5.50x + 24.50$ where $T(x)$ is the cost in dollars and $x$ is the number of miles the car is towed.

   a) Identify vertical intercept of this function. Write it as an ordered pair and explain its meaning in the context of this problem.

   b) Identify the slope of this function. Explain its meaning in the context of this problem.

21. Find the equation of the line passing through $(-3, 2)$ and $(5, 7)$. Leave your answer in $y = mx + b$ form.

22. Find the equation of the horizontal line passing through $(-3, 2)$.

23. Find the equation of the vertical line passing through $(-3, 2)$. 

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24. The function \( d(t) = -63.24t + 874.9 \) can be used to determine Donna’s distance from Phoenix as she is traveling home from her summer vacation in Idaho, after \( t \) hours of driving.

   a) Evaluate \( d(10) \) and interpret its meaning in the context of the problem.

   b) Find \( t \) so that \( d(t) = 0 \) and interpret it’s meaning in the context of the problem.

   c) Find the slope of the function and interpret its meaning in the context of the problem.

   d) Find the vertical intercept of the function and interpret its meaning in the context of the problem.

   e) Suppose Donna wants to make the trip home in 10 hours. How much faster would she need to travel? Explain.
Exponential Functions

25. Complete the following table. Use proper notation.

<table>
<thead>
<tr>
<th></th>
<th>( f(x) = 24(1.32)^x )</th>
<th>( f(x) = 3324(0.92)^x )</th>
<th>( f(x) = (1.04)^x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth or Decay?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth or Decay Rate (as a percent)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical Intercept</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal Intercept</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal Asymptote (equation)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
26. Determine if each data set is linear or exponential and write the formula for each. Show complete work.

a) 
\[
\begin{array}{cccccccc}
    x & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\
    f(x) & .04 & .2 & 1 & 5 & 25 & 125 & 625 \\
\end{array}
\]

b) 
\[
\begin{array}{cccccccc}
    x & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\
    f(x) & -1.375 & -1.5 & 3.75 & 12.5 & 21.25 & 3 & 3.875 \\
\end{array}
\]

c) 
\[
\begin{array}{cccccccc}
    x & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\
    f(x) & -3 & -5.5 & -8 & -10.5 & -13 & -15.5 & -18 \\
\end{array}
\]

d) 
\[
\begin{array}{cccccccc}
    x & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
    f(x) & 2 & 4 & 8 & 16 & 32 & 64 & 128 \\
\end{array}
\]
27. Fred and Wilma purchase a home for $150,000. Using function notation, write a formula for the value, \( V \), of the house \( t \) years after its purchase, assuming that the value

- a) Decreases by $1,000 per year.
- b) Decreases by 3% per year.
- c) Increases by $3,000 per year.
- d) Increases by 5% per year.

28. The function \( f(x) = 300(1.15)^x \) gives the population of a city (in thousands) \( x \) years since 2000.

- a) Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in this situation.

- b) Is the population of this city increasing or decreasing? At what rate? Write your answers in complete sentences and include all appropriate units.

- c) Determine \( f(10) \) and write a sentence explaining the meaning of your answer.

- d) How long will it take the population of this city to reach one million? Show all work and write your answer in a complete sentence.

- e) How long will it take the population to double? Show all work and write your answer in a complete sentence.
29. You purchased a vehicle for $30,000. Assuming the value of the car decreases at 8% per year, write an equation that represents the value, $V(t)$, of the car $t$ years from now. How many years does it take for the value to decay to $20,000? Round to the nearest hundredth.

30. Determine an exponential regression function, $P(t)$ to represent the data below. Let $t=0$ in the year 1930. Round “a” to the nearest whole number and “b” to 4 places. In what year will the population reach 5000 (round to the nearest whole year)?

<table>
<thead>
<tr>
<th>Year</th>
<th>1930</th>
<th>1940</th>
<th>1980</th>
<th>1990</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>25908</td>
<td>25622</td>
<td>19057</td>
<td>17533</td>
<td>16328</td>
</tr>
</tbody>
</table>
31. In each situation below, you will need to graph to find the solution to the equation using the INTERSECTION method. Fill in the missing information for each situation. Include a rough but accurate sketch of the graphs and intersection point. Mark and label the intersection. Round answers to two decimal places.

a) Solve $54(1.05)^x = 250$  
Solution: $x =$ ______________

\[
\begin{array}{|c|}
\hline
X_{\text{min}}: \quad \\
X_{\text{max}}: \quad \\
Y_{\text{min}}: \quad \\
Y_{\text{max}}: \quad \\
\hline
\end{array}
\]

b) Solve $2340(0.82)^x = 1250$  
Solution: $x =$ ______________

\[
\begin{array}{|c|}
\hline
X_{\text{min}}: \quad \\
X_{\text{max}}: \quad \\
Y_{\text{min}}: \quad \\
Y_{\text{max}}: \quad \\
\hline
\end{array}
\]
Logarithmic Functions

32. Complete the table.

<table>
<thead>
<tr>
<th>Exponential Form</th>
<th>Logarithmic Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $6^3 = 216$</td>
<td></td>
</tr>
<tr>
<td>b) $5^{-2} = \frac{1}{25}$</td>
<td></td>
</tr>
<tr>
<td>c) $8^0 = 1$</td>
<td></td>
</tr>
<tr>
<td>d) $\log_7 16807 = 5$</td>
<td></td>
</tr>
<tr>
<td>e) $\log x = 5$</td>
<td></td>
</tr>
</tbody>
</table>

33. Evaluate each of the following logarithms. Write “N” if the answer does not exist.

a) $\log_b 1 =$

b) $\log_b b =$

c) $\log_b 0 =$

d) $\log_b b^n =$

34. Evaluate each of the following logarithms without a calculator. Your answers must be exact.

a) $\log_5 1 =$

b) $\log_5 \left(\frac{1}{3}\right) =$

c) $\log_2 2 =$

d) $\log_8 (64) =$

e) $\log_5 \left(\frac{1}{25}\right) =$

f) $\log_3 \sqrt[3]{10} =$

35. Use the change of base formula and your calculator to evaluate each of the following.

Show your work. Round your answers to two decimal places.

a) $\log_5 (81)$

b) $\log_3 (57)$
36. Consider the function $g(x) = \log_3 x$;

a) Graph $g(x)$ on your graphing calculator. In the space below, draw what you see on your calculator screen. Use window $x_{\text{min}}=0$, $x_{\text{max}}=10$, $y_{\text{min}}=-2$, $y_{\text{max}}=2$.

   

b) What is the domain of $g(x)$?

c) What is the range of $g(x)$?

d) For what values of $x$ is $g(x)$ positive?

e) For what values of $x$ is $g(x)$ negative?

f) For what values of $x$ is $g(x)$ increasing?

g) What is the vertical intercept?

h) What is the horizontal intercept?

i) Give the equation of the vertical asymptote for $g(x)$.

j) For what value of $x$ is $g(x) = 1$?

k) For what value of $x$ is $g(x) = 3$?

l) Determine $g(42)$. Round your answer to three decimal places.

37. Evaluate $30 - 5\log_2 8$ both WITH and WITHOUT your calculator.
38. Solve the following equations. Simplify your answers. Where applicable, give both the exact answer and the decimal approximation rounded to three decimal places. Show all algebraic work. Do not round until the end of the problem!!

a) \( 8 - 2 \log_7 x = 10 \)  
b) \( 1000(1.12)^x = 2000 \)

c) \( 3 + \log(120 - x) = 5 \)  
d) \( 4^{2x} = 1000 \)

e) \( 2340(0.82)^x = 1250 \)  
f) \( 5 + 3 \log(x - 2) = 7 \)

g) \( 54(1.05)^x = 250 \)  
h) \( \log_7(5x - 1) = 10 \)
39. Fill out the following table. Intercepts must be written as ordered pairs. Always use proper notation. Round to two decimal places.

<table>
<thead>
<tr>
<th></th>
<th>( f(x) = x^2 - 5x + 4 )</th>
<th>( g(x) = 16 - x^2 )</th>
<th>( y = x^2 - 2x + 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opens Upward or Downward?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical Intercept</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal Intercept(s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertex</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axis of Symmetry (Equation)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

40. Factor each of the following. Write your answer in completely factored form.
a) \(3x^2 - 9x\)  

b) \(x^2 - 4x + 3\)  

c) \(x^2 + x - 30\)  

d) \(x^2 - 9\)

41. Solve \(x^2 + 18x - 68 = 20\) using the methods indicated below. Show all work.

a) Solve by graphing. Sketch the graph on a good viewing window (the vertex, intercepts and intersection points must appear on the screen). Mark and label the solutions on your graph.

Xmin: _______  Xmax: _______

Ymin: _______  Ymax: _______

Solution(s): __________________________

b) Solve by factoring.

c) Use the quadratic formula to solve.
42. Solve the following equations algebraically (Factoring or Quadratic Formula). You must show all algebraic steps for full credit. Where applicable, give both the exact answers and the decimal approximations rounded to three decimal places. Write complex solutions in the form \( x = a + bi \) and \( x = a - bi \) Use your calculator to check your answers. Sketch the graph on a good viewing window (the vertex, vertical intercept, and any horizontal intercepts should appear on the screen). Mark and label any real solutions on the graph.

a) \( 2x^2 - 8x + 10 = 4 \)

b) \( 2x^2 = -6x \)

c) \( x^2 + 4 = 4x \)
d) \( x^2 - 2x + 5 = 0 \)

e) \( x^2 - 3x = 10 \)

f) \( -x^2 + x = 2 \)
Lesson 12 – Course Review

Practice Problems

43. Simplify the following:
   a) \(3i(5 - 2i)\)  
b) \((3 + i) - (2 - 3i)\)  
c) \((3 + i)(2 - 3i)\)

44. Given a quadratic equation in the form \(f(x) = ax^2 + bx + c\), draw the graph of a parabola where \(a > 0\) and \(c < 0\).

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45. Suppose \( h(t) = -16t^2 + 40t + 80 \) represents the height of a ball (measured in feet above the ground) thrown from a roof as a function of time (in seconds).

a) Find the value(s) of \( t \) such that \( h(t) = 24 \). Interpret your results in the context of this problem.

b) Write the equation you would solve to determine when the ball will hit the ground. Solve this equation to an accuracy of two decimal places. Show your work.

c) Determine the maximum height of the ball. Explain how you found this.

d) Determine the practical domain and range of \( h(t) \)
### Radical Functions

46. Complete the table. Write intercepts as ordered pairs. Use inequality notation for domain and range. Round to the nearest hundredth as needed.

<table>
<thead>
<tr>
<th></th>
<th>$f(x) = \sqrt{3x}$</th>
<th>$f(x) = \sqrt{x + 9}$</th>
<th>$f(x) = \sqrt{12 - x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Intercept</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal Intercept</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Determine $f(5)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Determine $x$ when $f(x) = 3$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sketch the Graph on an appropriate viewing window. Label all intercepts and interesting features of the graph.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
47. Solve each of the equations algebraically. Show all of your work. Round any decimal results to three places. Write your answers in exact form.

a) \( 6 + \frac{3}{7} - 3x = 16 \)  
b) \( \sqrt{8x - 7} = x \)

c) \( 4\sqrt{x - 6} = 12 \)  
d) \( \frac{4}{3}2x + 8 + 5 = 0 \)

e) \( \sqrt{2x + 10} + 5 = x + 6 \)  
f) \( 5 - \frac{3}{5}x = 11 \)
### Rational Functions

48. Complete the table. Write intercepts as ordered pairs. Round to the nearest hundredth as needed.

<table>
<thead>
<tr>
<th></th>
<th>( f(x) = \frac{6}{3x} )</th>
<th>( f(x) = \frac{2x + 12}{4 - x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Intercept</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal Intercept</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical Asymptote</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(equation)</td>
<td></td>
<td></td>
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<tr>
<td>Horizontal Asymptote</td>
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<td>(equation)</td>
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<td>Determine ( f(5) )</td>
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<td>Determine ( x ) \text{ when } f(x) = 5</td>
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49. Solve the following equations algebraically. You must show all algebraic steps for full credit. Where applicable, give both the exact answers and the decimal approximations rounded to three decimal places. Use your calculator to check your answers. Sketch the graph on a good viewing window. Mark and label the solution(s) on the graph.

a) \( \frac{5}{9} = \frac{8}{x+1} \)

b) \( 4x = 7 + \frac{2}{x-1} \)