

# Unit 13: Polynomials and Exponents

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**Section 13.1:** Polynomials

**Section 13.2:** Operations on Polynomials

**Section 13.3:** Properties of Exponents

**Section 13.4:** Multiplication of Polynomials

**Section 13.5:** Applications from Geometry

**Section 13.6:** Division Properties of Exponents

KEY TERMS AND CONCEPTS	
Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms <i>in your own words</i> . Provide examples that are not identical to those in the Media Lesson.	
Polynomial	
Monomial	
Binomial	
Trinomial	
Leading Term	

Leading Coefficient	
Degree of a Polynomial	
Constant Term	
The Multiplication Property	
Raising a Power to a Power	
Raising a Product to a power	
The Division Property	
Raising a Quotient to a Power	

# Unit 13: Media Lesson

## Section 13.1: Polynomials

Definitions
<b>Polynomial:</b> An algebraic expression composed of the sum of terms containing a single variable raised to a non-negative integer exponent.
<b>Monomial:</b> A polynomial consisting of <b>one</b> term
<b>Binomial:</b> A polynomial consisting of <b>two</b> terms
<b>Trinomial:</b> A polynomial consisting of <b>three</b> terms
<b>Leading Term:</b> The term that contains the highest power of the variable in a polynomial
<b>Leading Coefficient:</b> The coefficient of the leading term
<b>Constant Term:</b> A number with no variable factors. A term whose value never changes.
<b>Degree:</b> The highest exponent in a polynomial

 **Example 1:** Complete the table.

Polynomial	Name	Leading Coefficient	Constant Term	Degree
$24a^6 + a^2 + 5$				
$2m^3 + m^2 - 2m - 8$				
$5x^2 + x^3 - 7$				
$-2x + 4$				
$4x^3$				

## Section 13.1 – You Try




Complete the table.


Polynomial	Name	Leading Coefficient	Constant Term	Degree
$n^2 - 2n + 8$				
$4x^3$				
$6x - 7$				

## Section 13.2: Operations on Polynomials


### Addition of Polynomials

 **Example 1:** Add.  $(3n^2 - 2n + 8) + (3n^3 - 7n^2 - n - 9)$

### Subtraction of Polynomials

 **Example 2:** Subtract.  $(a^3 + 5a + 11) - (4a^3 + 6a^2 - a + 1)$

### Combine and Simplify

 **Example 3:** Perform the indicated operations. Simplify.

$$(3x - 1) - (x^2 - x - 9) + (4x^3 + x^2 - 7x + 2)$$

## Section 13.2 – YOU TRY



Perform the indicated operations. Simplify completely. Show all steps as in the media examples.

a.  $(x^2 - x + 8) + (5x^2 - 6x - 11)$

b.  $(8x^2 - 4x + 5) - (3x^2 - 4x + 6)$

c.  $(5x + 8) + (x^2 - x - 1) - (x^3 + 3x^2 - 4x + 8)$

## Section 13.3: Properties of Exponents

Given any real numbers  $a, b, c, m,$  and  $n$

$n^1 = \underline{\hspace{2cm}}$

$1^n = \underline{\hspace{2cm}}$

$n^0 = \frac{\hspace{2cm}}{n \neq 0}$

$0^n = \frac{\hspace{2cm}}{n \neq 0}$

$3^4 = \underline{\hspace{2cm}}$

$3^3 = \underline{\hspace{2cm}}$

$3^2 = \underline{\hspace{2cm}}$

$3^1 = \underline{\hspace{2cm}}$

$3^0 = \underline{\hspace{2cm}}$

$3^{-1} = \underline{\hspace{2cm}}$

$3^{-2} = \underline{\hspace{2cm}}$

$3^{-3} = \underline{\hspace{2cm}}$

$3^{-4} = \underline{\hspace{2cm}}$

Multiplication Properties of Exponents	
$a^m \cdot a^n = a^{m+n}$ <p style="text-align: center;">Why?</p>	$(a^m)^n = a^{mn}$ <p style="text-align: center;">Why?</p>



**Example 1:** Evaluate and simplify the following expressions.  
Assume  $x \neq 0$ ,  $x \neq -1/2$ ,  $a \neq 0$ ,  $b \neq 0$ , and  $c \neq 0$ .

$5x^0$

$(2x + 1)^0$

$a^0 + b^0 + c^0$

The Multiplication Property:  $a^m \cdot a^n = a^{m+n}$




**Example 2:** Simplify the following expressions

$n^3 n^9$

$b^5 \cdot b^4 \cdot b$

$5x^2 y^5 (7xy^9)$


Raising a Power to a Power:  $(a^m)^n = a^{mn}$

 **Example 3:** Simplify the following expressions

$$(x^3)^9$$

$$5b^2(b^5)^8$$

Raising a Product to a Power:  $(ab)^n = a^n b^n$

 **Example 4:** Simplify the following expressions

$$(5x)^2$$


$$(x^3y^2)^9$$

$$(-8ab^5)^2$$

$$5(-2w^7)^3$$

$$5n^4(-3n^3)^2$$

Section 13.3 – You Try

 Simplify the following expressions. Show all steps as in the media examples.

a.  $(2x^4)^2$

b.  $2(x^2)^3$

c.  $8g^3 \cdot 5g^4$

d.  $2n^0$



## Section 13.4: Multiplication of Polynomials

### Multiplication of Monomials

 **Example 1:** Multiply and simplify.

$$(3x^5)(-2x^9)$$

### The Distributive Property

 **Example 2:** Expand and simplify.

$$5x^3(2x^5 - 4x^3 - x + 8)$$

### Multiplication of Polynomials

 **Example 3:** Multiply and simplify.

a.  $(x + 3)(x + 4)$

b.  $(m - 5)(m - 6)$

c.  $(2d - 4)(3d + 5)$

d.  $(x - 2)(x^2 + 2x - 4)$

## Squaring a Binomial

**Example 4:** Multiply and simplify

a.  $(n + 5)^2$

b.  $(3 - 2a)^2$

## Section 13.4 – You Try



Multiply and simplify. Show all steps as in the media examples.

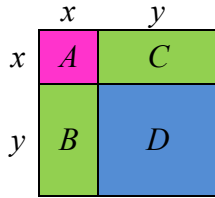
a.  $-3x^2(x^5 + 6x^3 - 5x)$


b.  $(3x - 4)(5x + 2)$

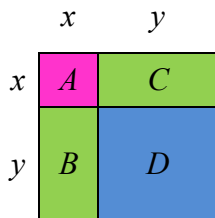
c.  $(2p - 5)^2$


## Section 13.5: Applications from Geometry

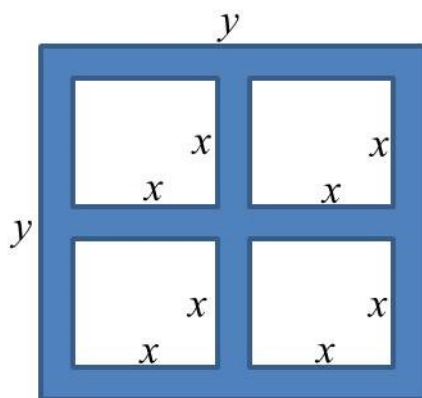
 **Example 1:** Write a polynomial in simplest form that represents the area of the square.



 **Example 1 (another way):** Write a polynomial in simplest form that represents the area of the square.



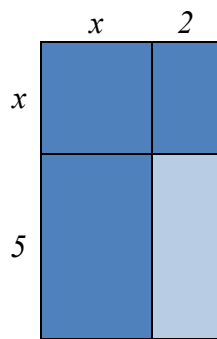
 **Example 2:** Write a polynomial in simplest form that represents the area of the shaded region.



## Section 13.5 – You Try



Write a polynomial in simplest form that represents the area of the dark blue region of the figure shown below. Show all steps as in the media examples.



## Section 13.6: Division Properties of Exponents

$$\text{The Division Property: } \frac{a^m}{a^n} = a^{m-n} \quad a \neq 0$$

$$\frac{x^5}{x^2} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = \frac{x \cdot x \cdot x}{1} = \frac{x^3}{1} = x^3$$

$$\frac{x^5}{x^2} = x^{5-2} = x^3$$

 **Example 1:** Simplify the following expressions. Variables represent nonzero quantities.

$$\frac{x^{50}}{x^4}$$

$$\frac{4a^{10}b^5}{6ab^2}$$

$$\text{Raising a Quotient to a Power: } \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad b \neq 0$$

 **Example 2:** Simplify the following expressions. Variables represent nonzero quantities.

$$\left(\frac{5}{7}\right)^2$$

$$\left(\frac{x^5}{y^3}\right)^4$$

$$\left(\frac{-4t^{10}}{u^6}\right)^2$$

## Section 13.6 – You Try



Simplify the following expressions. Variables represent nonzero quantities. Show all steps as in the media examples.

a.  $\left(\frac{3a^{10}}{7}\right)^2$

b.  $\frac{6x^3y^8}{9xy^5}$

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Unit 13: Practice Problems

### Skills Practice

1. Complete the table below.

Polynomial	Name	Leading Coefficient	Constant Term	Degree
$5n^8 - n^5 + 1$				
$x - 5$				
$8r^2$				

2. Simplify completely. Show all steps, and box your answers.

a.  $(2x)^3$

b.  $5(3n)^2$

c.  $y^3 \cdot y^7 \cdot y$

d.  $(-2x)^3$

e.  $5w(8w^3)$

f.  $(-2x^5)^2$

g.  $(-5w^8)^2$

h.  $3x^0 + 2x^0$

i.  $(-4x)^2 + 4x^2$

j.  $(5x - 7)^0$

3. Multiply and simplify completely. Show all steps, and box your answers.

a.  $4x^2(3x - 5)$

b.  $4a^2(3a^2 - 2a - 5)$

c.  $(p + 5)(p + 7)$

d.  $(x + 2)(x - 2)$

e.  $(2x - 4)(3x - 5)$

f.  $(5w - 8)(3w + 11)$



g.  $(x + 2)^2$

h.  $(2x - 4)^2$

i.  $(x - 4)(x^2 + x - 5)$

j.  $3(x + 2)(x + 4)$

k.  $4(x + 2)^2$

l.  $(q - 2)^3$

4. Simplify completely. Show all steps, and box your answers.

a.  $\frac{x^8}{x^3}$

b.  $\left(\frac{2}{5}\right)^4$

c.  $\frac{8n^8p^5}{12np^4}$

d.  $\left(\frac{3a^5}{7b}\right)^2$

5. Evaluate the algebraic expression  $x^2$  given  $x = -7$ . Show your work.

6. Evaluate the algebraic expression  $5x^3$  given  $x = -2$ . Show your work.

7. Evaluate the algebraic expression  $(5x)^2$  given  $x = -2$ . Show your work.

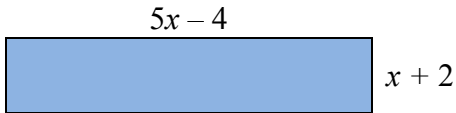
8. Evaluate the algebraic expression  $5(2x)^2$  given  $x = -3$ . Show your work.

9. Evaluate the algebraic expression  $\frac{6}{5x}$  given  $x = -2$ . Show your work.

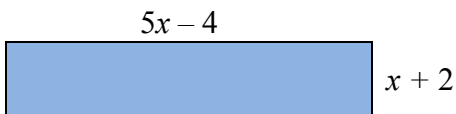
10. Evaluate the algebraic expression  $\frac{1}{4x^2}$  given  $x = -5$ . Show your work.

## Applications

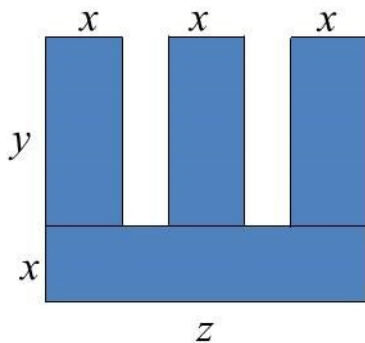
11. Write an algebraic expression that represents the perimeter of the figure shown below. Simplify completely. Show your work.



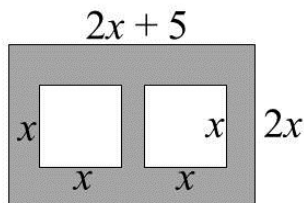
12. Write an algebraic expression that represents the total area of the figure shown below. Simplify completely. Show your work.



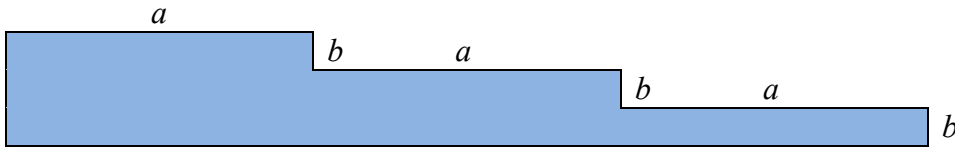
13. Write an expression that represents the total area of the figure shown below. Simplify completely.



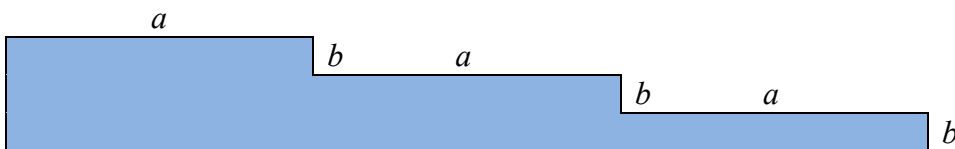
14. Write an expression that represents the area of the shaded region of the figure shown below. Simplify completely.



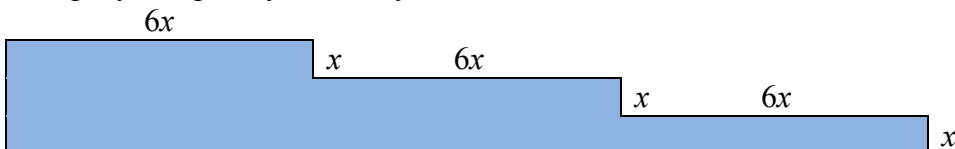
15. Write an expression that represents the total area of the figure shown below. Simplify completely.



16. Write an expression that represents the perimeter of the figure shown below. Simplify completely.



17. Write an algebraic expression that represents the total area of the figure shown below. Simplify completely. Show your work.



## Extension

18. If possible, simplify each of the following by combining like terms or using properties of exponents.

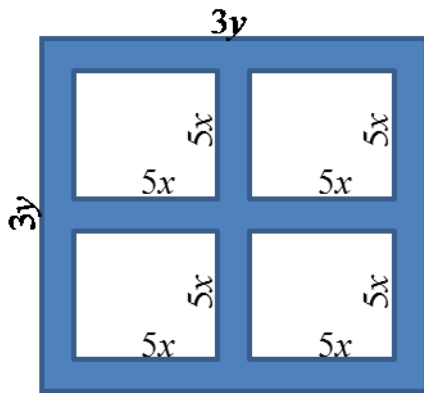
a.  $2n^5 + 3n^5 =$  \_\_\_\_\_

b.  $2n^5 \cdot 3n^5 =$  \_\_\_\_\_

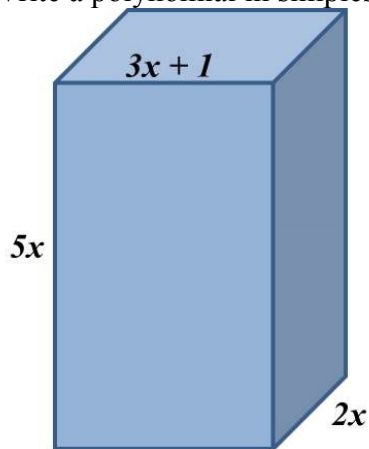
c.  $3n^3 + 3n^5 =$  \_\_\_\_\_

d.  $3n^3 \cdot 3n^5 =$  \_\_\_\_\_

19. Write an algebraic expression that represents the total area of the figure shown below. Simplify completely. Show your work.



20. Write a polynomial in simplest form that represents the volume of the figure shown below.



21. Simplify completely. Show all steps, and box your answers.

a.  $4p(-5p^3)^2$

b.  $3(-2x)^3 - 3x(-2)^3$

c.  $4w^5(3w^8)^2$

d.  $10p^3(-5p^7)^2$

e.  $2a^3b(3ab^5)^2$

f.  $(3x^4)^3 - (5x^6)^2$

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Unit 13: Review

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1. Consider the polynomial  $n^2 - 7n - 11$

a. Is this a monomial, binomial, or trinomial? \_\_\_\_\_

b. Identify the constant term. \_\_\_\_\_

c. What is the leading coefficient? \_\_\_\_\_

d. What is the degree of this polynomial? \_\_\_\_\_

e. Identify the coefficient of the second term. \_\_\_\_\_

2. If possible, simplify each of the following by combining like terms or using properties of exponents.

a.  $8n^3 + 5n^3 =$  \_\_\_\_\_

b.  $8n^3 \cdot 5n^3 =$  \_\_\_\_\_

c.  $8n^3 + 8n^5 =$  \_\_\_\_\_

d.  $8n^3 \cdot 8n^5 =$  \_\_\_\_\_

3. Simplify completely. Show all steps, and box your answers.

a.  $(-5x^5)^3$

b.  $4x^2(8x^2 - 5x - 3)$

c.  $(3 - 5x)^2$

d.  $\frac{24m^8}{18m^3}$

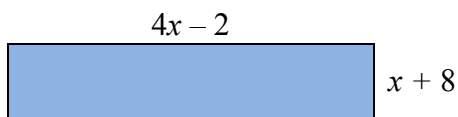
e.  $\left(\frac{5x}{3}\right)^2$

f.  $(4x^3 + 6x^2 - x + 7) - (3x^3 + x^2 - x - 5)$

4. Evaluate the algebraic expression  $8(2x)^2$  given  $x = -5$ . Show your work.

5. Evaluate the algebraic expression  $\frac{2}{3x^2}$  given  $x = 4$ . Show your work.

6. Write an algebraic expression that represents the perimeter of the figure shown below. Simplify completely. Show your work.



7. Write an algebraic expression that represents the total area of the figure shown below. Simplify completely. Show your work.

