

Unit 12: Systems of Equations

Section 12.1: Systems of Linear Equations

Section 12.2: The Substitution Method

Section 12.3: The Addition (Elimination) Method

Section 12.4: Applications

KEY TERMS AND CONCEPTS	
Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms <i>in your own words</i> . Provide examples that are not identical to those in the Media Lesson.	
System of Linear Equations	
Solution to a System of Linear Equations	
Types of Solutions to a System of Linear Equations	

Substitution Method	
Addition (Elimination) Method	

Unit 12: Media Lesson

Section 12.1: Systems of Linear Equations

Definitions

Two linear equations that relate the same two variables are called a **system of linear equations**. A **solution** to a system of linear equations is an **ordered pair** that satisfies both equations.



Example 1: Verify that the point (5, 4) is a solution to the system of equations

$$y = 2x - 6$$

$$y = x - 1$$

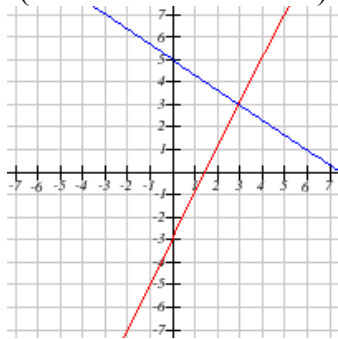
Types of Solutions to a Linear System of Equations

Graphically, the solution to a system of linear equations is a point at which the graphs intersect.

Types of Solutions to a Linear System of Equations:

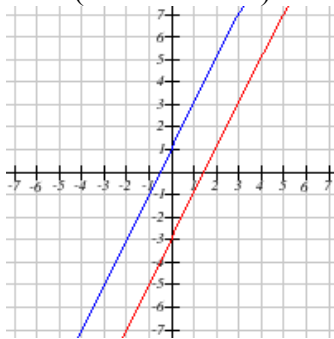
- **One unique solution:** The lines intersect at exactly one point
- **No solution:** The two lines are parallel and will never intersect
- **Infinitely many solutions:** This occurs when both lines graph as the same line

One Unique Solution
(One Intersection Point)



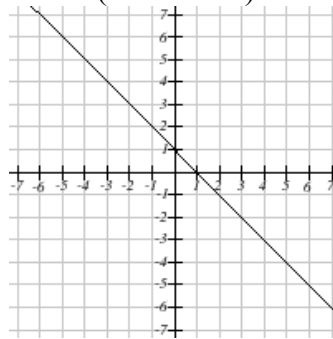
Consistent and Independent

No Solution
(Parallel Lines)



Inconsistent

Infinitely Many Solutions
(Same Line)



Consistent and Dependent

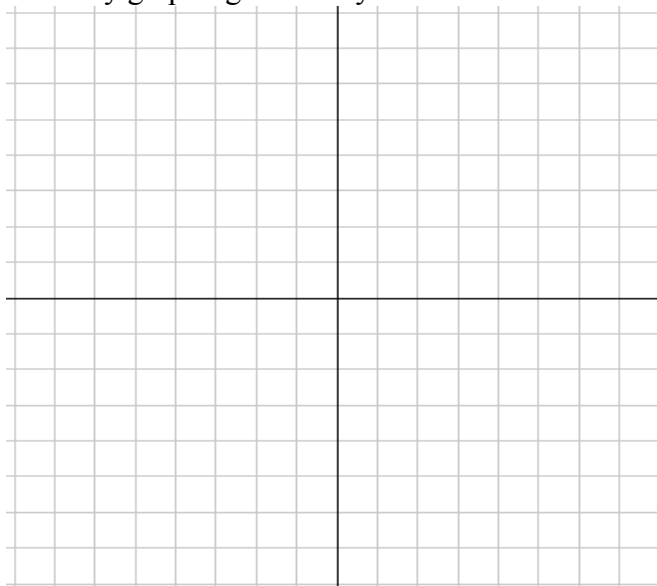
Solving a System of Linear Equations by Graphing



Example 2: Solve the system of equations by graphing. Check your answer.

$$y = 6 - \frac{2}{3}x$$

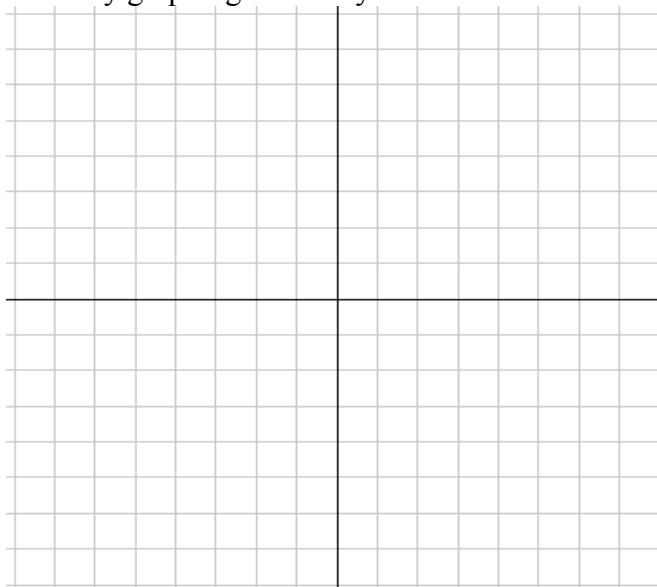
$$y = x + 1$$




Example 3: Solve the system of equations by graphing. Check your answer.

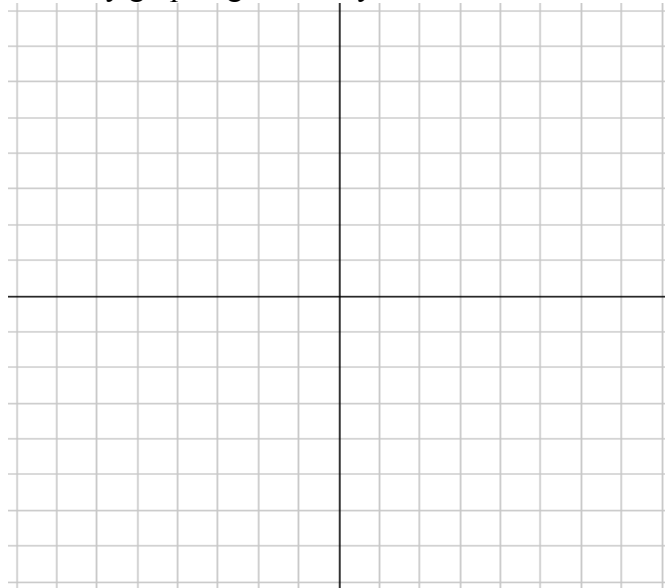
$$4x - 3y = -18$$


$$2x + y = -4$$



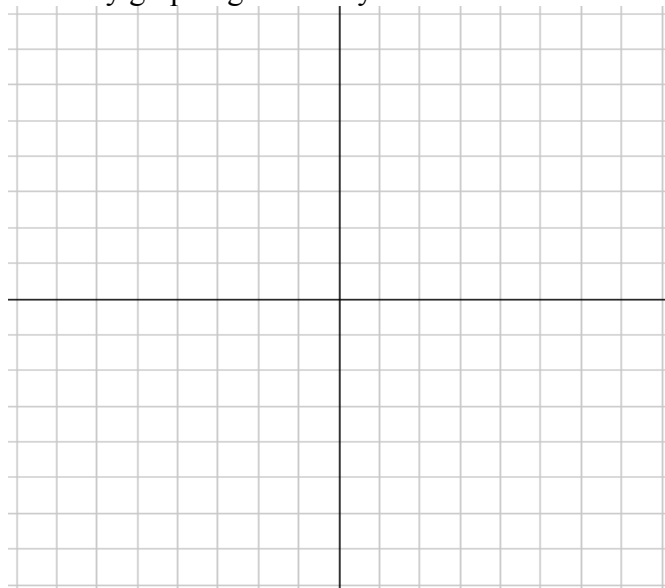
 **Example 4:** Solve the system of equations by graphing. Check your answer.

$$\begin{aligned}x - 3y &= 3 \\ 3x - 9y &= -18\end{aligned}$$



 **Example 5:** Solve the system of equations by graphing. Check your answer.

$$\begin{aligned}2x + y &= 3 \\ 6x + 3y &= 9\end{aligned}$$



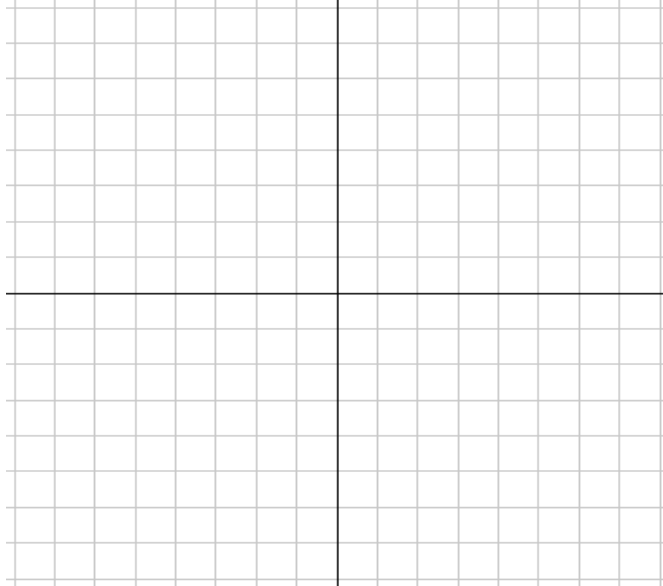
Section 12.1 – You Try



Solve the system of equations by graphing. Write your answer as an ordered pair and verify that it is correct.

$$x - y = 2$$

$$x + y = 6$$



Verify that your solution is correct:

Section 12.2: The Substitution Method

Consider the following equations: $y = 2x$
 $x + y = 3$

Using Substitution to Solve a Linear System of Equations

Step 1: Solve one of the equations of the system for one of the variables.

Step 2: Substitute the expression for the variable obtained in step 1 into the other equation.

Step 3: Solve the equation.

Step 4: Substitute the result back into one of the original equations to find the ordered pair solution.

Step 5: Check your result by substituting your result into either one of the original equations.



Example 1: Solve the system of equations using the Substitution Method.

$$3x - 2y = 16$$

$$2x + y = 20$$



Example 2: Solve the system of equations using the Substitution Method.

$$5x - 4y = 9$$

$$x - 2y = -3$$



Example 3: Solve the system of equations using the Substitution Method.

$$3x + y = 5$$

$$6x + 2y = 11$$



Example 4: Solve the system of equations using the Substitution Method.

$$x - y = -1$$

$$y = x + 1$$

Section 12.2 – You Try



Solve the system of equations using the Substitution Method. Show all steps. Check your answer.

$$x - 2y = -11$$

$$5x + 2y = 5$$

Section 12.3: The Addition (Elimination) Method

Consider the following systems of equations:

$$\begin{aligned}x - 2y &= -11 \\ 5x + 2y &= 5\end{aligned}$$

Using the Addition (Elimination) Method to Solve a Linear System of Equations

Step 1: “Line up” the variables.

Step 2: Determine which variable you want to eliminate. Make those coefficients opposites.

Step 3: Add straight down (one variable should “drop out”)

Step 4: Solve resulting equation

Step 5: Substitute this result into either of the ORIGINAL equations

Step 6: Solve for the variable

Step 7: CHECK!!!!!! Plug solution into BOTH equations!



Example 1: Solve the system of equations using the Addition (Elimination) Method.

$$4x - 3y = -15$$

$$x + 5y = 2$$



Example 2: Solve the system of equations using the Addition (Elimination) Method.

$$3x - 2y = -12$$

$$5x - 8y = 8$$



Example 3: Solve the system of equations using the Addition (Elimination) Method.

$$7x - 2y = 41$$

$$3x - 5y = 1$$

Section 12.3 – You Try



Solve the system of equations using the Addition (Elimination) Method. Show all steps.
Check your answer.

$$2x + 3y = 18$$

$$x - y = 4$$

Section 12.4 – You Try



Tickets to a 3D movie cost \$12.50 for adults and \$8.50 for children. The theater can seat up to 180 people. A total of \$1,826 was collected in ticket sales for the sold-out 7:15PM show. Determine the number of adult tickets and the number of children's tickets that were sold.

- a. Write an equation representing the total number of tickets sold. Clearly indicate what each variable represents.

- b. Write an equation representing the total amount of money collected from the sale of all tickets.

- c. Solve this system of linear equations. Show all steps.

Number of adult tickets sold: _____

Number of children's tickets sold: _____

Unit 12: Practice Problems

Skills Practice

1. Is the point $(6, 1)$ a solution to the system of equations below? You must show correct work to justify your answer.

$$\begin{aligned}y &= x - 5 \\y &= 2x + 4\end{aligned}$$

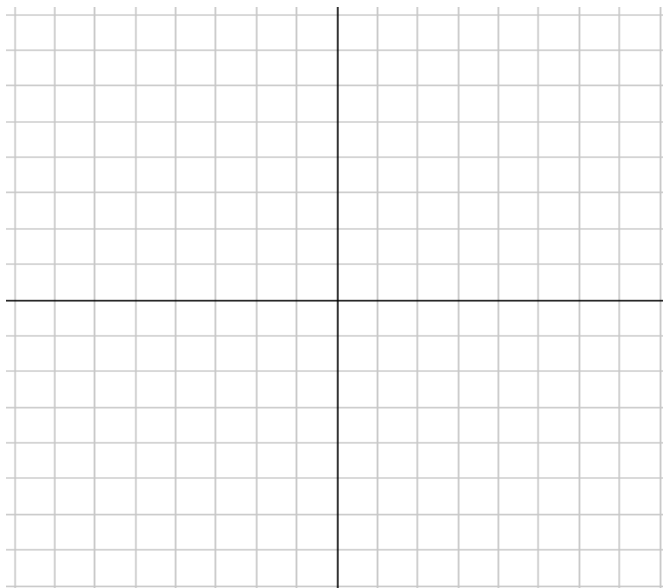
2. Is the point $(-2, 5)$ a solution to the system of equations below? You must show correct work to justify your answer.

$$\begin{aligned}2x + y &= 1 \\3x - 2y &= -16\end{aligned}$$

3. Is the point $(5, 3)$ a solution to the system of equations below? You must show correct work to justify your answer.

$$\begin{aligned}3x - 2y &= 9 \\2x + 5y &= 4\end{aligned}$$

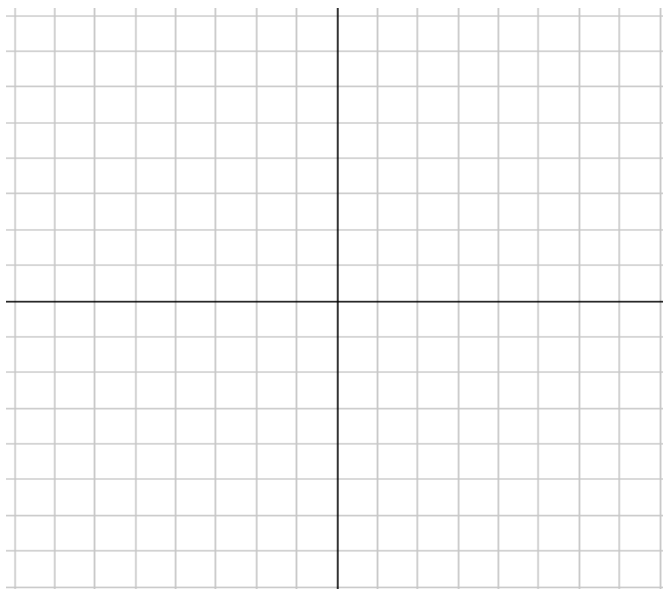
4. Solve the system of equations by **graphing**. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.



$$y = 7 - x$$
$$y = 3x - 5$$

Solution: _____

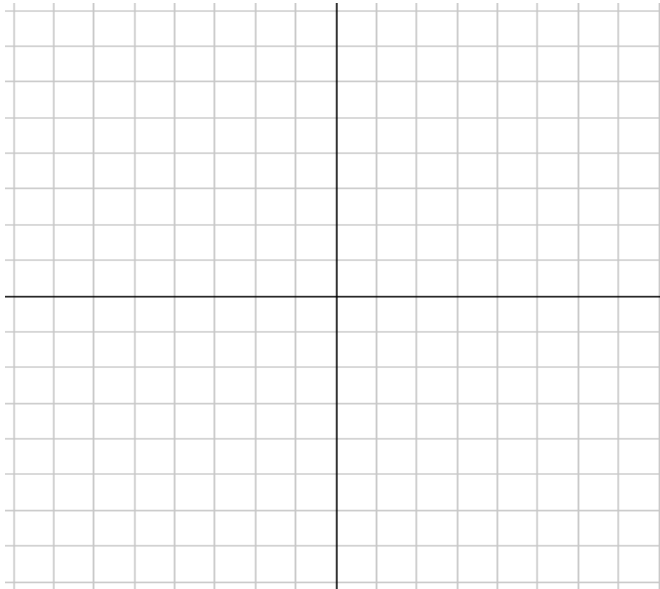
5. Solve the system of equations by **graphing**. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.



$$x - y = -2$$
$$x + y = 4$$

Solution: _____

6. Solve the system of equations by **graphing**. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.

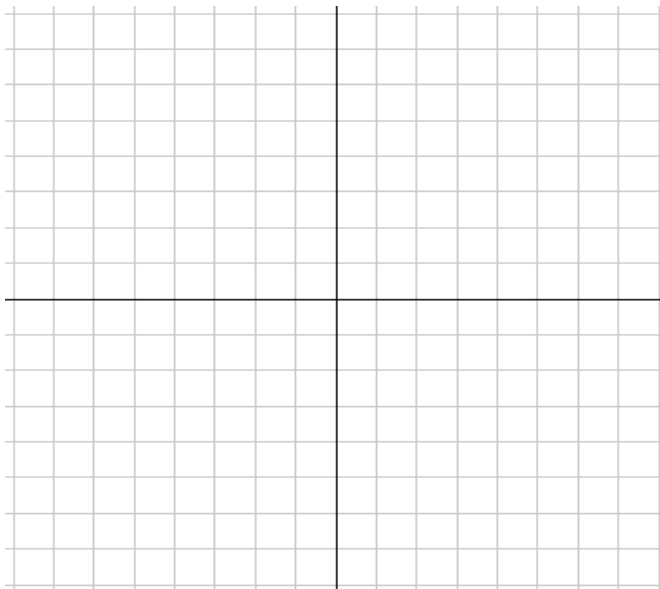


$$x - 2y = 10$$

$$5x - y = -4$$

Solution: _____

7. Solve the system of equations by **graphing**. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.

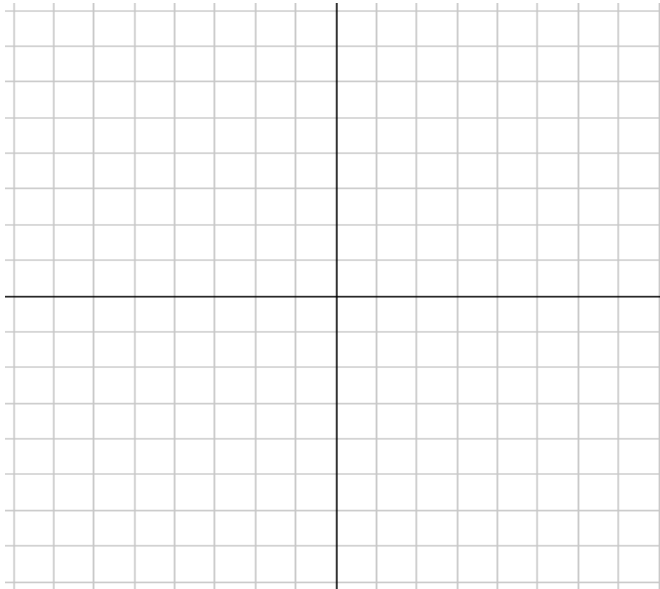


$$3x - y = 8$$

$$-3x + y = 1$$

Solution: _____

8. Solve the system of equations by **graphing**. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.



$$\begin{aligned}x + 2y &= -4 \\2x + 4y &= -8\end{aligned}$$

Solution: _____

9. Solve the system of equations using the **substitution** method. Show all steps.

$$\begin{aligned}5x + y &= 2 \\3x - 4y &= 15\end{aligned}$$

Solution: _____

10. Solve the system of equations using the **substitution** method. Show all steps.

$$2x + y = 8$$

$$6x + 3y = 24$$

Solution: _____

11. Solve the system of equations using the **substitution** method. Show all steps.

$$x - y = 9$$

$$5x + 3y = 21$$

Solution: _____

12. Solve the system of equations using the **addition (elimination) method**. Show all steps.

$$-3x + 2y = 12$$

$$x + y = 16$$

Solution: _____

13. Solve the system of equations using the **addition (elimination) method**. Show all steps.

$$3x - 2y = -12$$

$$12x - 8y = 22$$

Solution: _____

14. Solve the system of equations using the **addition (elimination) method**. Show all steps.

$$3x + 2y = -18$$

$$4x - 3y = -24$$

Solution: _____

15. Solve the system of equations using the **addition (elimination) method**. Show all steps.

$$5x + 2y = -10$$

$$3x + 4y = 8$$

Solution: _____

16. The functions $f(x)$ and $g(x)$ are defined by the following tables. At what point is $f(x) = g(x)$?

x	-2	-1	0	1	2	3	4
$f(x)$	11	8	5	2	-1	-4	-7

x	-2	-1	0	1	2	3	4
$g(x)$	7	6	5	4	3	2	1

Solution (write the ordered pair): _____

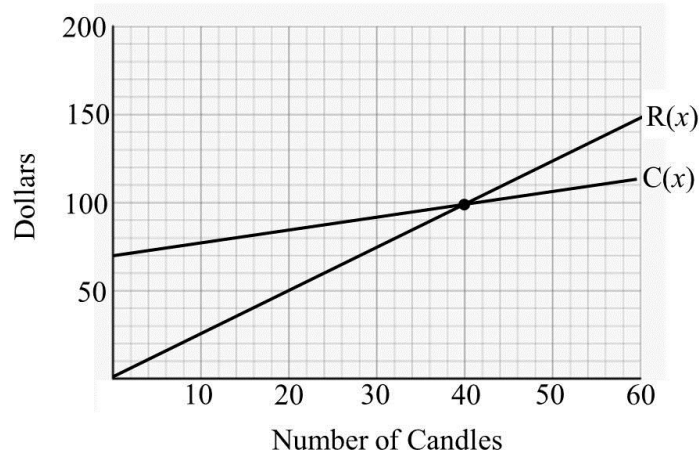
17. The functions $f(x)$ and $g(x)$ are defined by the following tables. At what point is $f(x) = g(x)$?

x	-2	-1	0	1	2	3	4
$f(x)$	8	1	0	-1	-8	-27	-64

x	-2	-1	0	1	2	3	4
$g(x)$	8	10	12	14	16	18	20

Solution (write the ordered pair): _____

19. The graph below shows the cost and revenue for a company that produces and sells scented candles. The function $R(x)$ gives the revenue earned when x candles are sold. The function $C(x)$ gives the total cost to produce x candles.



- Discuss the significance of the point $(40, 100)$ in terms of the cost, revenue, and *profit* for this company.
 - What happens if *fewer than* 40 candles are sold?
 - What happens if *more than* 40 candles are sold?
20. At a concession stand, five hot dogs and five sodas cost \$30. Two hot dogs and four sodas cost \$15. Determine the price of each hot dog and each soda.

Price for each soda: _____

Price for each hot dog: _____

21. The Science Museum charges \$14 for adult admission and \$11 for each child. The total bill for 68 people from a school field trip was \$784. How many adults and how many children went to the museum?

Number of children _____

Number of adults _____

22. Tickets to a 3D movie cost \$12.50 for adults and \$8.50 for children. The theater can seat up to 260 people. A total of \$1,734 was collected in ticket sales for the 7:15PM show, in which only 60% of the tickets were sold. How many adults and how many children were in the theater?

Number of children _____

Number of adults _____

23. Jake has 20 coins in his pocket, all of which are dimes and quarters. If the total value of his change is \$4.10, how many dimes and how many quarters does he have?

Number of dimes _____

Number of quarters _____

24. Juan had \$17400 and chose to split the money into two different mutual funds. During the first year, Fund A earned 3% interest and Fund B earned 6% interest. If he received a total of \$774 in interest, how much had he invested into each account?

Amount invested in Fund A: _____

Amount invested in Fund B: _____

25. Emery invested \$10,000 in two mutual funds. Fund A earned 4% profit during the first year, while Fund B suffered a 2% loss. If she received a total of \$130 profit, how much had she invested in each mutual fund?

Amount invested in Fund A: _____

Amount invested in Fund B: _____

26. Bill begins a 100 mile bicycle ride. Unfortunately, his bicycle chain breaks, and he is forced to walk the rest of the way. The whole trip takes 6 hours. If Bill walks at a rate of 4 miles per hour, and rides his bike at a rate of 20 miles per hour, find the amount of time he spent walking. Write your answer in a complete sentence. (Hint: Distance = rate · time)

Extension

27. The functions $f(x)$ and $g(x)$ are defined by the following tables.

At what point(s) is $f(x) = g(x)$?

x	-2	-1	0	1	2	3	4
$f(x)$	4	1	0	1	4	9	16

x	-2	-1	0	1	2	3	4
$g(x)$	-1	1	3	5	7	9	11

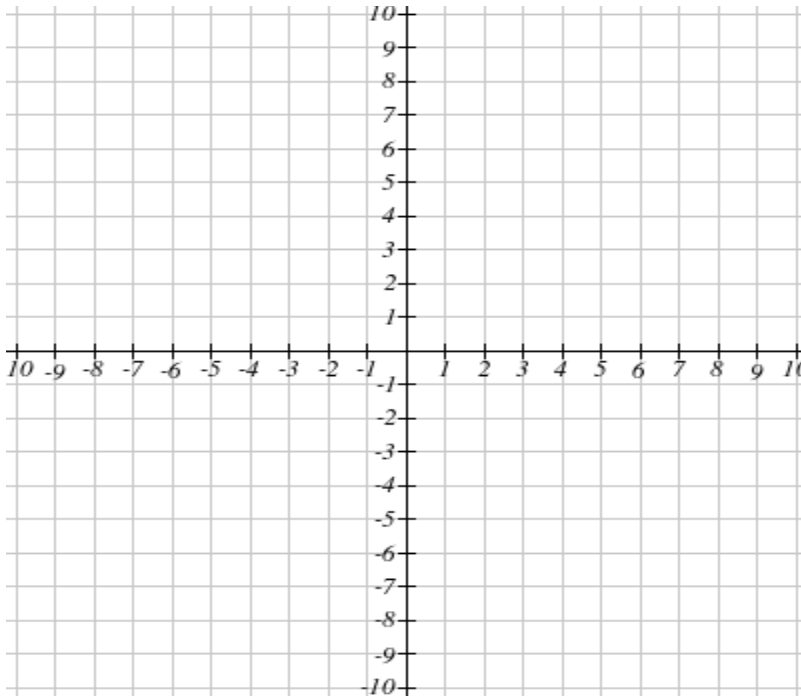
Solutions (write the ordered pairs): _____

28. Construct a system of linear equations (in slope-intercept form) that has the ordered pair (3,5) as a solution.

29. Construct a system of linear equations (in general form) that has the ordered pair (2,4) as a solution.

Unit 12: Review

1. Solve the system of equations by **graphing**. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.



$$\begin{aligned}4x - 3y &= -18 \\ 3x + y &= -7\end{aligned}$$

Solution: _____

2. Solve the system of equations using the **substitution** method. Show all steps. Verify that your solution is correct.

$$\begin{aligned}2x - 3y &= -19 \\ x + 2y &= 8\end{aligned}$$

Solution: _____

3. Solve the system of equations using the **addition (elimination)** method. Show all steps. Verify that your solution is correct.

$$5x - 2y = -3$$

$$7x - y = 12$$

Solution: _____

4. Jamaal invested \$10,000 in two mutual funds. Fund A earned 6% profit during the first year, and Fund B earned 2% profit. If he received a total of \$374 profit, how much had he invested in each mutual fund? Show all steps. Verify that your solution is correct.

Amount invested in Fund A: _____

Amount invested in Fund B: _____
