Unit 7: Introduction to Functions

Section 7.1: Relations and Functions

Section 7.2: Function Notation

Section 7.3: Domain and Range

Section 7.4: Practical Domain and Range

Section 7.5: Applications

KEY TERMS AND CONCEPTS	
Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms *in your own words*. Provide examples that are not identical to those in the Media Lesson.	
Relation	
Function	
Vertical Line Test	
Dependent Variable	
Independent Variable	

Behavior of Functions	
Function Notation	
Compare: Find $f(4)$ Find x when $f(x) = 4$	
Domain	
Range	
Practical Domain	
Practical Range	

Unit 7: Media Lesson

Section 7.1: Relations and Functions

Definitions

A **RELATION** is any set of ordered pairs.

A **FUNCTION** is a relation in which **every** input value is paired with **exactly one** output value

Table of Values

One way to represent the relationship between the input and output variables in a relation or function is by means of a table of values.

Example 1: Which of the following tables represent functions?

Input	Output
1	5
2	5
3	5
4	5

Yes No

Input	Output
1	8
2	–9
3	7
3	12

Yes No

Input	Output
2	4
1	–5
4	10
–3	–87

Yes No

Ordered Pairs

A relations and functions can also be represented as a set of points or ordered pairs.

Example 2: Which of the following sets of ordered pairs represent functions?

A = {(0, –2), (1,4), (–3,3), (5,0)}

B = {(–4,0), (2, –3), (2, –5)}

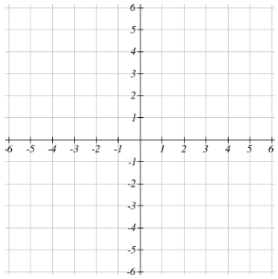
C = {(–5,1), (2,1), (–3,1), (0,1)}

D = {(3, –4), (3, –2), (0, 1), (2, –1)}

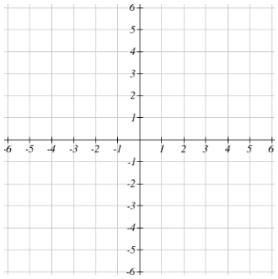
E = {(1,3)}

Example 3: On the graphs below, plot the points for A, B, C, and D from Example 2, then circle the "problem points"

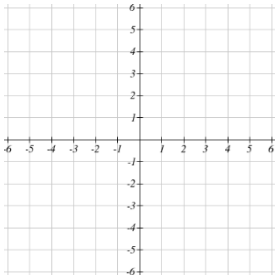
A



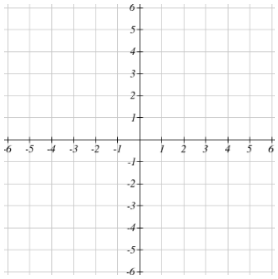
B



C

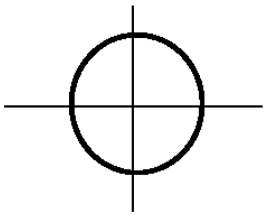


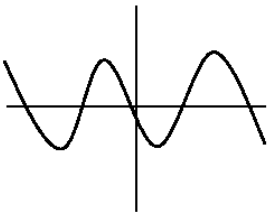
D

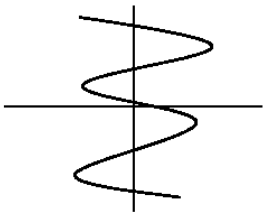


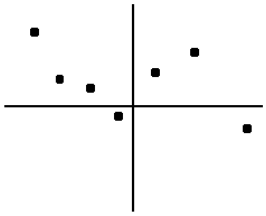
The Vertical Line Test
• If all vertical lines intersect the graph of a relation at no more than one point, the relation *is* also a function. One and only one output value exists for each input value. • If any vertical line intersects the graph of a relation at more than one point, the relation "fails" the test and is NOT a function. More than one output value exists for some (or all) input value(s).

Example 4: Use the Vertical Line Test to determine which of the following graphs are functions.









Behavior of Graphs		
Increasing	Decreasing	Constant

Dependent and Independent Variables

In general, we say that the output **depends** on the input.

Output variable = **Dependent Variable**

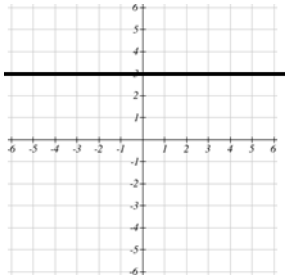
Input Variable = **Independent Variable**

If the relation is a function, then we say that the output **is a function of** the input.

Section 7.1 – You Try

Is it a function? Circle "Yes" or "No" for each of the following.

Yes or No



Yes or No

Input	Output
4	12
6	14
8	14
10	16

Yes or No

(2, –3), (–5, 2), (–3, 1)

Section 7.2: Function Notation: f(input) = output

If a relation is a function, we say that the *output is a function of the input.*

Function Notation: f(input) = output

Example: If y is a function of x, then we can write $f(x) = y$.

Example 1: The function V(m) represents value of an investment (in thousands of dollars) after m months. Explain the meaning of V(36) = 17.4.

Ordered Pairs

Example 2:

Ordered Pair (input, output)	Function Notation f(input) = output
(2, 3)	$f(2) = 3$
(–4, 6)	f(____) = _____
(_____ , _____)	$f(5) = -1$

Example 3: Consider the function: f = {(2, –4), (5, 7), (8, 0), (11, 23)}

$f(5)$ = _____ f(_____) = 0

Table of Values

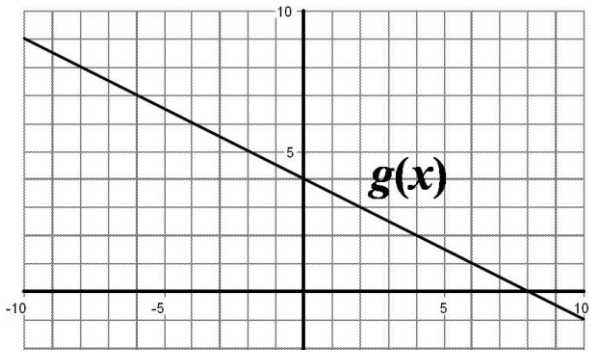
Example 4: The function $B(t)$ is defined by the table below.

t	1	3	12	18	22	31
$B(t)$	70	64	50	39	25	18

$B(12)$ = _____ $B(t) = 18$ when t = _____

Graph

Example 5: Consider the graph $g(x)$ of shown below



$g(2)$ = _____ g(_____) = 2

Ordered pair:_____ Ordered pair:_____

$g(0)$ = _____ g(_____) = 1

Ordered pair:_____ Ordered pair:_____

Section 7.2 –You Try

Complete the problems below.

a. Complete the table.

Ordered Pair	Function Notation
(8, 1)	$f($ ____ $) =$ _____
(_____ , _____)	$f(0) = 11$

b. The function $k(x)$ is defined by the following table

x	–2	–1	0	1	2	3	4
$k(x)$	8	2	–9	4	6	1	0

$k(2) =$ _____ $k(x) = 1$ when $x =$ _____

Ordered Pair: _____ Ordered Pair: _____

c. At an ice cream factory, the total cost production is a function of the number of gallons of ice cream produced. The function C(g), gives the cost, in dollars, to produce g gallons of ice cream. Explain the meaning of $C(580)=126$ in terms of ice cream production.

Section 7.3: Domain and Range

DEFINITIONS
The **DOMAIN** of a function is the set of all possible values for the **input** variable. The **RANGE** of a function is the set of all possible values for the **output** variable.

DOMAIN AND RANGE

Example 1: Consider the function below

x	–2	0	2	4	6
$k(x)$	3	–7	11	3	8

Input values _____

Domain: {_____}

Output values:_____

Range: {_____}

Example 2: Consider the function: $B = \{(2, -4), (5, 7), (8, 0), (11, 23)\}$

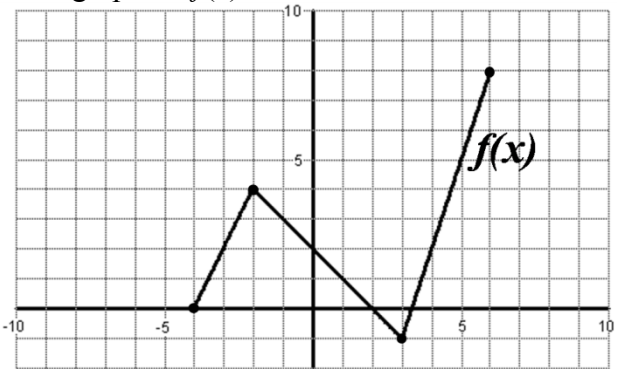
Input values _____

Domain: {_____}

Output values:_____

Range: {_____}

Example 3: Consider the graph of $f(x)$ shown below

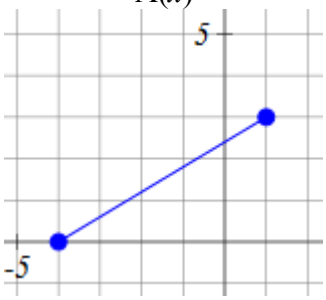


Domain: _____ $\leq x \leq$ _____

Range: _____ $\leq f(x) \leq$ _____

Example 4: Determine the Domain and Range of each of the following graphs:

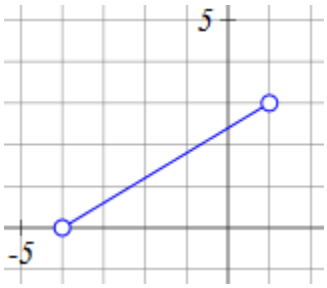
A(x)



Domain

Range

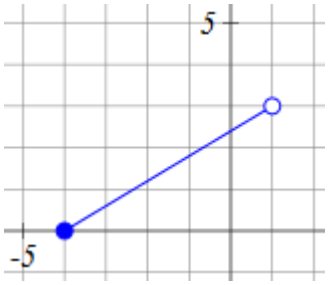
B(x)



Domain

Range

C(x)



Domain

Range

SECTION 7.3 – YOU TRY

Determine the Domain and Range of the functions below.

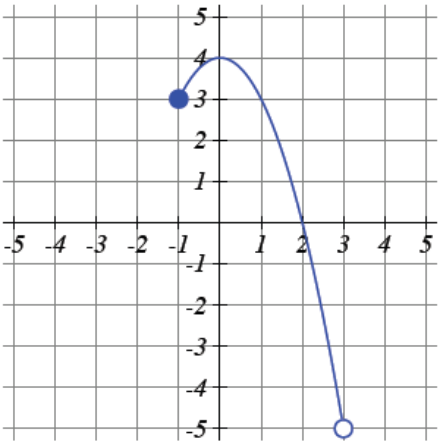
a.

Input	Output
4	12
6	12
8	12
10	12

Domain:

Range:

b. The graph of $f(x)$ is shown below



Domain:

Range:

Section 7.4: Practical Domain and Range

Definitions
The **Practical Domain** of a function is the set of all possible values for the input variable *that make sense* in a given situation. The **Practical Range** of a function is the set of all possible values for the output variable *that make sense* in a given situation.

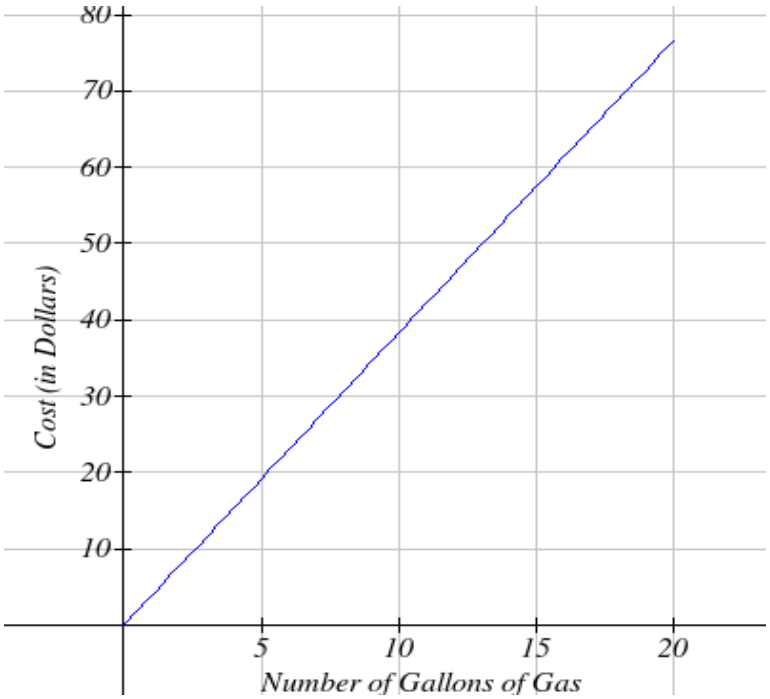
Example 1: The gas station is currently charging $3.83 per gallon for gas. The cost, C(n), in dollars, to fill up your car depends on the number of gallons, n, that you pump. Your car's tank can hold a maximum of 20 gallons of gas.

a. In this situation, the input variable is _____.

b. The *practical* domain of this function is _____.

c. The output variable in this situation is _____.

d. The *practical* range of this function is _____.



Section 7.4 – You Try

The platform for the high dive is 35 feet above the water. A diver jumps from the platform and lands in the water after 1.5 seconds. The function H(s) represents the height of the diver after s seconds.

a. In this situation, the input variable is _____.

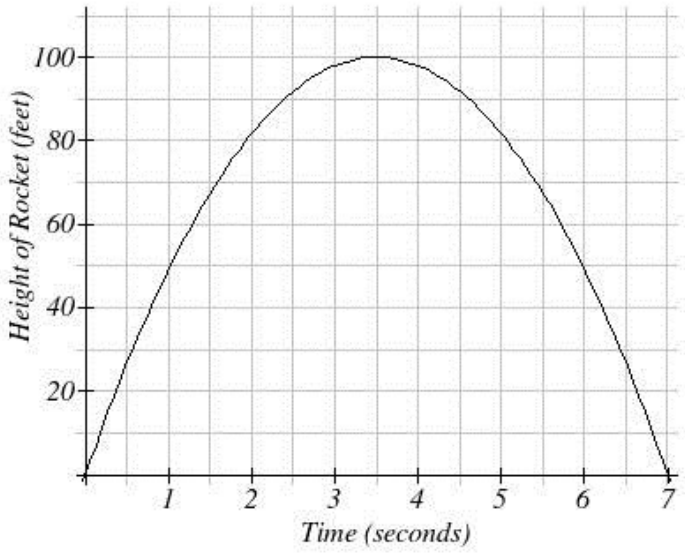
b. The *practical* domain of this function is _____.

c. The output variable in this situation is _____.

d. The *practical* range of this function is _____.

Section 7.5: Applications

Example 1: Consider the graph of the function H(t) shown below.



Input Variable: _____

Units of Input Variable: _____

Output Variable: _____

Units of Output Variable: _____

a. Interpret the meaning of the statement H(5)=82.

b. Determine H(7). Write it as an ordered pair and interpret its meaning in a complete sentence.

c. Determine t when H(t) = 50. Write it as an ordered pair and interpret its meaning in a complete sentence.

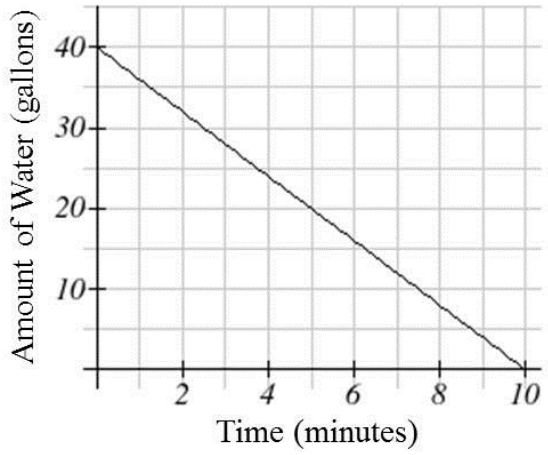
d. Determine the maximum height of the rocket.

e. Determine the practical domain for H(t).

f. Determine the practical range for H(t).

Section 7.5 – You Try

The graph of A(m) below shows the amount of water in a play pool.



Input Variable: _____

Units of Input Variable: _____

Output Variable: _____

Units of Output Variable: _____

a. Interpret the meaning of the statement A(3)=28.

b. Determine A(5). Write it as an ordered pair and interpret its meaning in a complete sentence.

c. Determine t when A(m) = 0. Write it as an ordered pair and interpret its meaning in a complete sentence.

d. Describe what is happening to the water in the pool. (Is the pool being filled or drained?)

e. Determine the practical domain for A(m). Use inequality notation and include units.

f. Determine the practical range for A(m). Use inequality notation and include units.

Unit 7: Practice Problems

Skills Practice

1. Are these functions? Circle yes or no.

Input	Output
3	12
7	12
4	12
2	12

Yes No

Input	Output
1	8
2	–9
3	7
3	12

Yes No

Input	Output
2	4
1	–5
4	10
–3	–87

Yes No

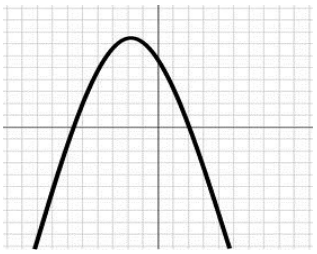
2. Are these functions? Circle yes or no.

   a. {(2, –4), (6, –4), (0, 0), (5, 0)} Yes No

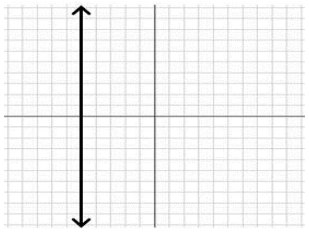
   b. {(1, 1), (2, 2), (3, 3), (4, 4)} Yes No

   c. {(1, –8), (5, 2), (1, 6), (7, –3)} Yes No

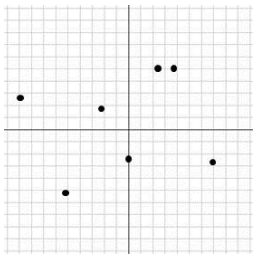
3. Are these functions? Circle yes or no.



Yes **No**



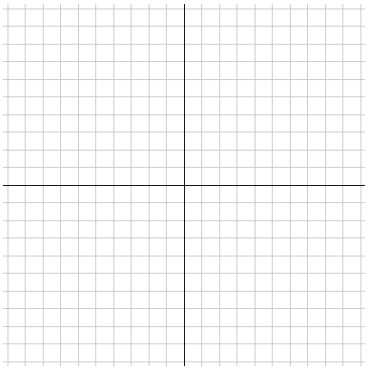
Yes **No**



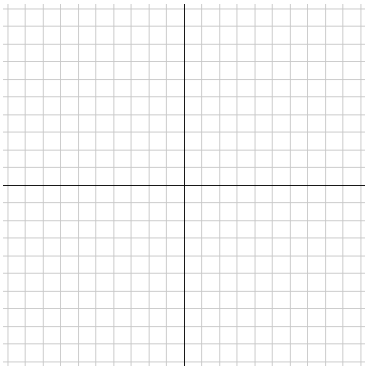
Yes **No**

4. In the space below, draw a graph that represents a function, and a graph that does NOT represent a function.

Function



Not a Function



5. The function $r(x)$ is defined by the following table of values.

x	3	5	6	9	13
$r(x)$	–9	3	2	2	1

a. $r(9) =$ _____ b. $r(3) =$ _____

c. $r($ _____ $) = 1$ d. $r($ _____ $) = 3$

e. The domain of $r(x)$ is { _____ }

f. The range of $r(x)$ is { _____ }

6. Consider the function $g = \{(2, 5), (0, 6), (5, 8), (–3, 7)\}$

a. $g(0) =$ _____ b. $g(5) =$ _____

c. $g($ _____ $) = 7$ d. $g($ _____ $) = 5$

e. The domain of g is { _____ }

f. The range of g is { _____ }

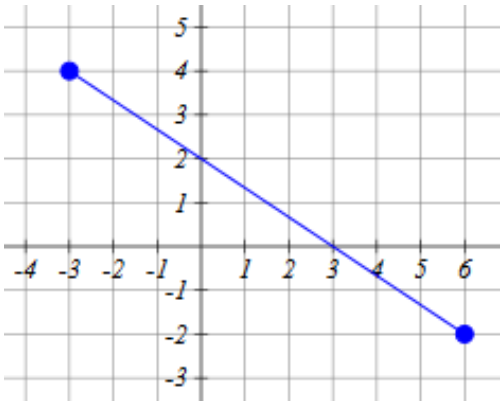
7. Given $f(4) = 8, f(3) = 11, f(0) = 6$

a. The domain of f is { _____ }

b. The range of f is { _____ }

c. Write the function f as a set of ordered pairs.

8. The graph of $g(r)$ is given below.



a. Domain: _____

b. Range _____

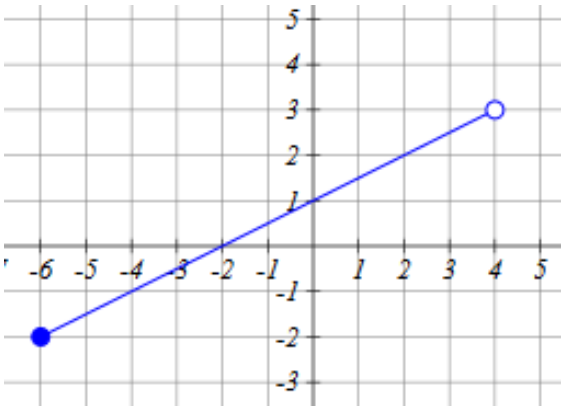
c. $g(-3) =$ _____

d. $g(0) =$ _____

e. $g(r) = 4$ when $r =$ _____

f. $g(r) = 0$ when $r =$ _____

9. The graph of $A(m)$ is given below.



a. Domain: _____

b. Range _____

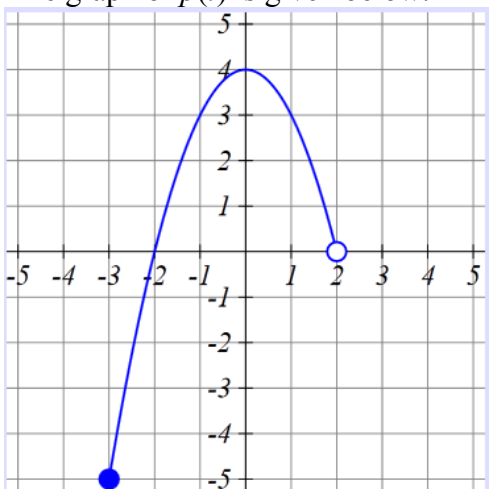
c. $A(-4) =$ _____

d. $A(0) =$ _____

e. $A(m) = -2$ when $m =$ _____

f. $A(m) = 0$ when $m =$ _____

10. The graph of $p(t)$ is given below.



a. Domain: _____

b. Range _____

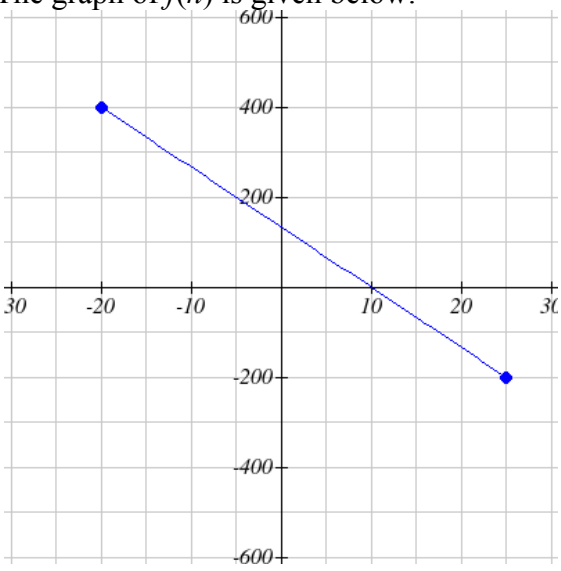
c. $p(-1) =$ _____

d. $p(0) =$ _____

e. $p(t) = -5$ when $t =$ _____

f. $p(t) = 3$ when $t =$ _____

11. The graph of $f(n)$ is given below.



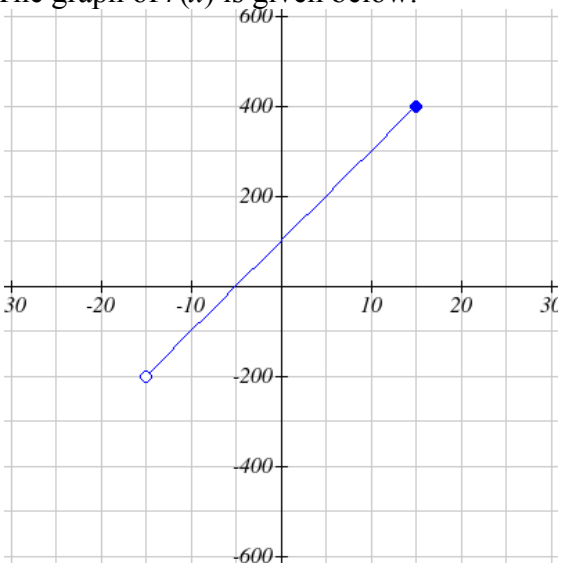
a. Domain: _____

b. Range _____

c. $f(-5) =$ _____

d. $f(n) = 0$ when $n =$ _____

12. The graph of $r(x)$ is given below.



a. Domain: _____

b. Range _____

c. $r(-10) =$ _____

d. $r(x) = 300$ when $x =$ _____

Applications

13. A rock is dropped from the top of a building. The function $H(t)$ gives the height (measured in meters) of the rock after t seconds. In a complete sentence, explain the meaning of the statement $H(2) = 35$. Your answer must include correct units.

14. The function $P(n)$ represents a computer manufacturer's profit, in dollars, when n computers are sold. In a complete sentence, explain the meaning of the statement $P(40) = 1680$. Your answer must include correct units.

15. The function $E(t)$ gives the surface elevation (in feet above sea level) of Lake Powell t years after 1999. In a complete sentence, explain the meaning of the statement $E(5) = 3675$. Your answer must include correct units.

16. The function $V(n)$ gives the value, in thousands of dollars, of an investment after n months. In a complete sentence, explain the meaning of the statement $V(24) = 18$. Your answer must include correct units.

17. The function $P(t)$ can be used to approximate the population of a town, in thousands of people, t years after 1980. In a complete sentence, explain the meaning of the statement $P(31) = 52$. Your answer must include correct units.

18. A candy company has a machine that produces candy canes. The table below is a partial list of the relationship between the number of minutes the machine is operating and the number of candy canes produced by the machine during that time period.

Minutes t	3	5	8	12	15
Candy Canes $C(t)$	12	20	32	48	60

a. Include units. $C(12) =$ _____

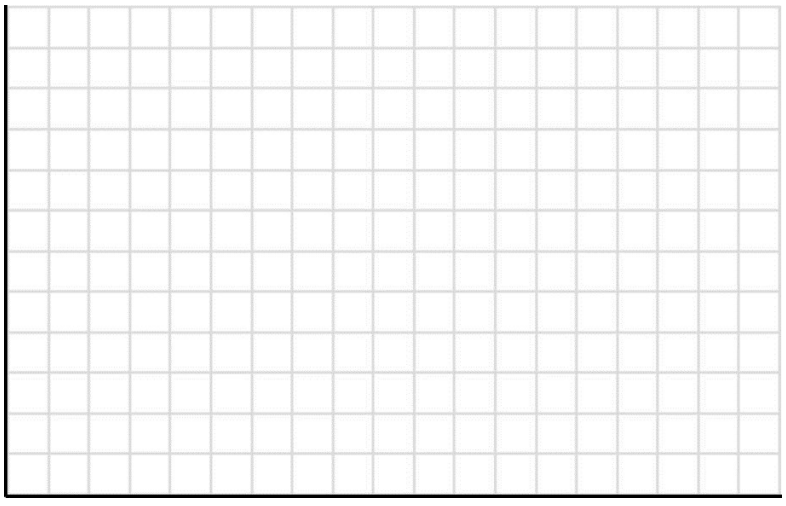
b. In a complete sentence and including all appropriate units, explain the meaning of your answer in part a.

c. Include units. $C(t) = 12$ when $t =$ _____

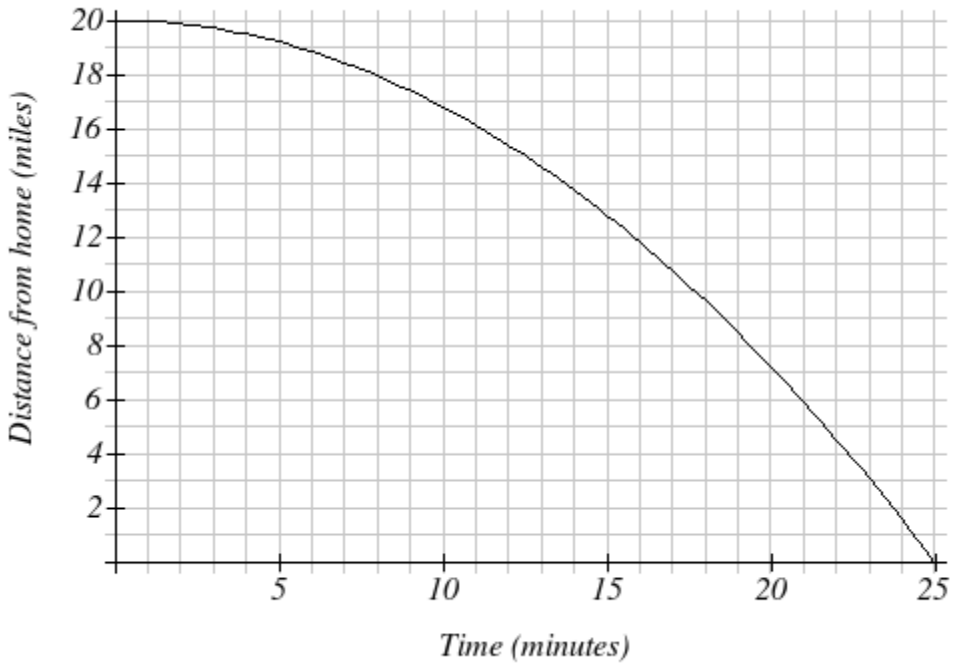
d. In a complete sentence and including all appropriate units, explain the meaning of your answer in part c.

e. This function is (circle one) **increasing** **decreasing**

f. Construct a properly scaled and labeled graph $C(t)$.



19. The function D(t) is shown below.



a. Determine $D(0)$ and interpret its meaning in a complete sentence.

b. Determine $D(8)$ and interpret its meaning in a complete sentence.

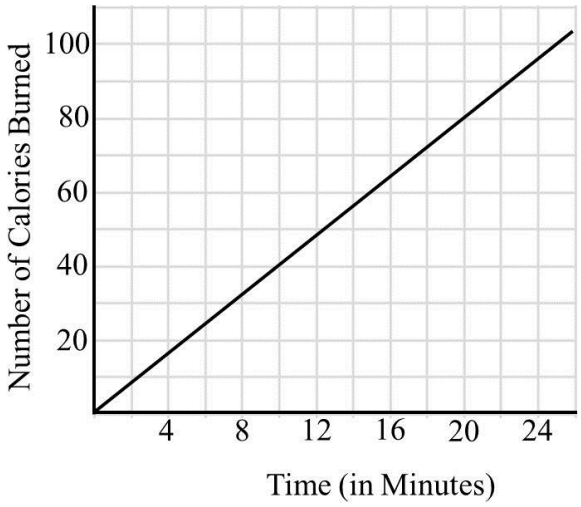
c. For what value of t is $D(t) = 3$? Write a sentence explaining the meaning of your answer.

d. For what value of t is $D(t) = 0$? Write a sentence explaining the meaning of your answer.

e. Determine the practical domain of $D(t)$.

f. Determine the practical range of $D(t)$.

20. The graph of the function $C(n)$ below shows the number of calories burned after riding a stationary bike for n minutes.



a. Is this function increasing or decreasing? _____

b. Interpret the meaning of the statement $C(8) = 32$.

c. Determine $C(10)$ and interpret its meaning in a complete sentence.

d. For what value of n is $C(n) = 80$? Write a sentence explaining the meaning of your answer.

Extension

21. Sort the following terms into the two groups below.

Dependent Variable

Domain

Horizontal Axis

Independent variable

Range

Vertical Axis

Input	Output

22. In a relation, we say that the output **depends** on the input. If the relation is a function, then we say that the output **is a function of** the input. For each of the following, identify the input variable and the output variable, and then determine if the relation is a function.

a. Is the outside temperature in Tempe, AZ a function of the time of day?

Input Variable: _____

Output Variable: _____

Function? Yes No

b. Is your letter grade a function of your numerical grade in the class?

Input Variable: _____

Output Variable: _____

Function? Yes No

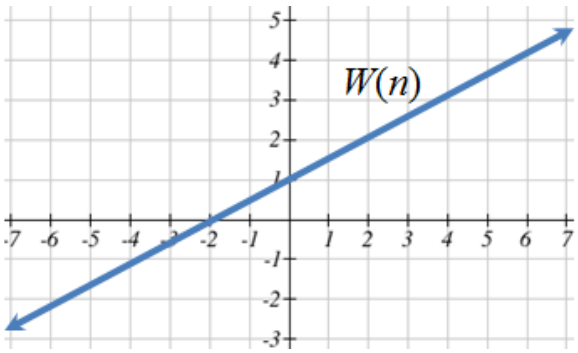
c. Is your numerical grade a function of your letter grade?

Input Variable: _____

Output Variable: _____

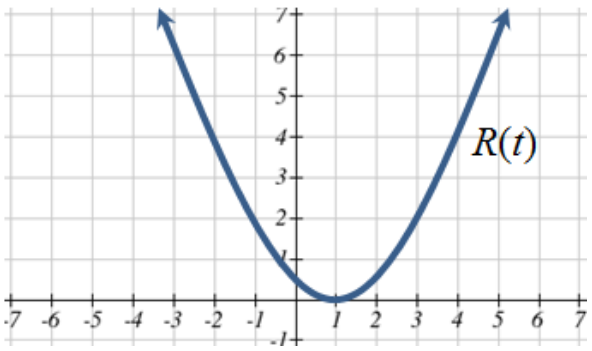
Function? Yes No

23. Determine the domain and range of each of the graphs shown below. Use correct notation.



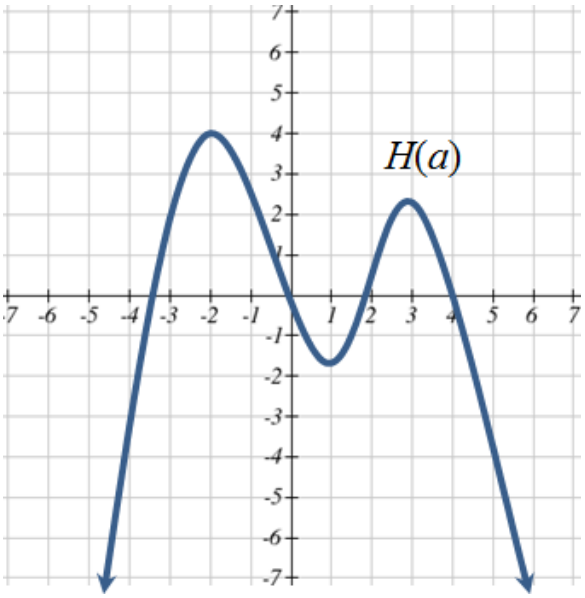
Domain: _____

Range: _____



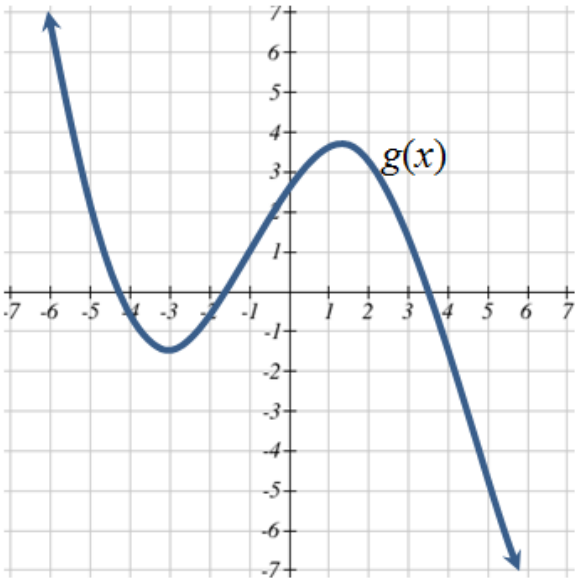
Domain: _____

Range: _____



Domain: _____

Range: _____



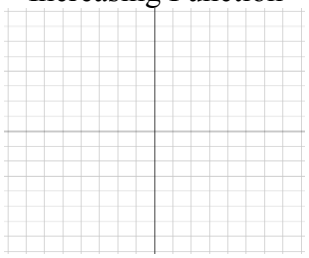
Domain: _____

Range: _____

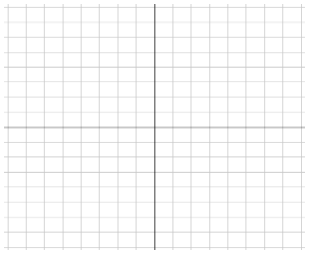
Unit 7: Review

1. In the space below, draw a graph that represents an increasing function, a constant function, and a graph that does NOT represent a function.

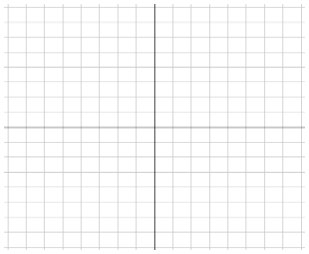
Increasing Function



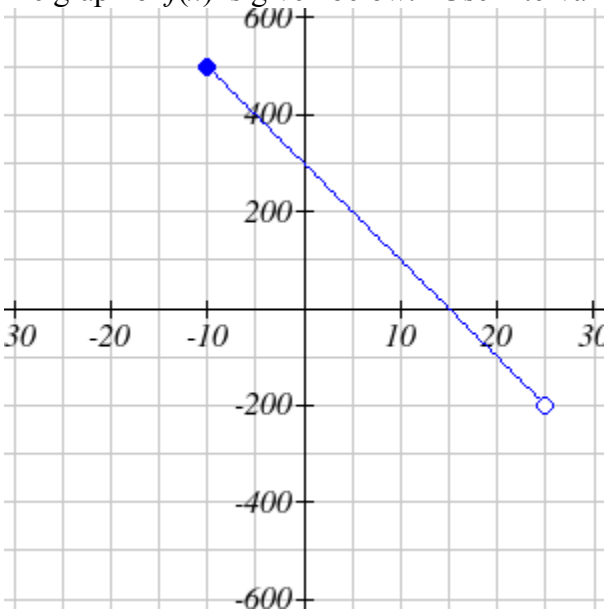
Constant Function



Not a Function



2. The graph of $f(x)$ is given below. Use interval notation for the domain and range.



a) Domain: _____

b) Range _____

c) $f(0) =$ _____

d) $f(x) = 0$ when $x =$ _____

3. Consider the following table of values. Fill in the blanks below, and identify the corresponding ordered pairs.

x	–2	–1	0	1	2	3	4
$g(x)$	1	4	8	6	5	0	2

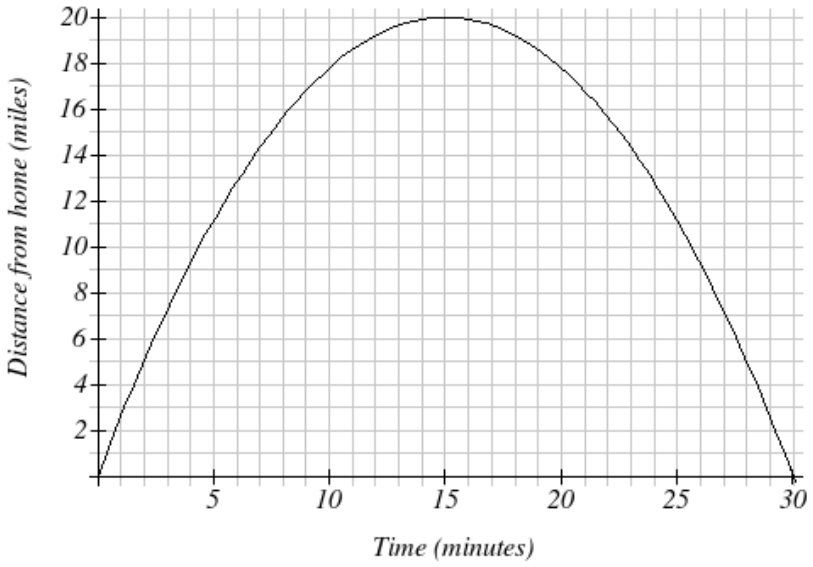
$g(1) =$ _____

Ordered pair: _____

$g(x) = 1$ when $x =$ _____

Ordered Pair: _____

4. The function D(t) shown below represents Sally's distance from home over a 30-minute time period.



a. Identify the vertical intercept of D(t). Write it as an ordered pair and explain its meaning in this situation.

b. Identify the horizontal intercepts of D(t). Write them as an ordered pairs and explain their meaning in this situation.

c. Determine $D(15)$ and interpret its meaning in a complete sentence.

d. For what value of t is $D(t) = 5$? Write a sentence explaining the meaning of your answer.

e. Determine the practical domain of $D(t)$. _____

f. Determine the practical range of $D(t)$. _____