Unit 1: Introduction to Variables

Section 1.1: Writing Algebraic Expressions

Section 1.2: The Story of "*x*"

Section 1.3: Evaluating Algebraic Expressions

Section 1.4: Applications

Section 1.5: Geometric Formulas

KEY TERMS AND CONCEPTS	
Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms *in your own words*. Provide examples that are not identical to those in the Media Lesson.	
Variable	
Algebraic Expression	
Evaluate an Algebraic Expression	
The Story of "*x*"	

Commutative Property	
Exact Form	
Approximate Form	

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Unit 1: Media Lesson

Section 1.1: Writing Algebraic Expressions

Definitions

A **variable**, usually represented by a letter or symbol, can be defined as:

- A quantity that may change within the context of a mathematical problem.
- A placeholder for a specific value.

An **algebraic expression** is a mathematical statement that can contain numbers, variables, and operations (addition, subtraction, multiplication, division, etc…).

 **Example 1:** Juan is 6 inches taller than Niko. Let N represent Niko's height in inches. Write an algebraic expression to represent Juan's height.

 **Example 2:** Juan is 6 inches taller than Niko. Let J represent Juan's height in inches. Write an algebraic expression to represent Niko's height.

 **Example 3:** Suppose sales tax in your town is currently 9.8%. Write an algebraic expression representing the sales tax for an item that costs D dollars.

 **Example 4:** You started this year with $362 saved and you continue to save an additional $30 per month. Write an algebraic expression to represent the total amount saved after *m* months.

 **Example 5:** Movie tickets cost $8 for adults and $5.50 for children. Write an algebraic expression to represent the total cost for *A* adults and *C* children to go to a movie.

Section 1.1 – You Try

 Complete the following problems. Show all steps as in the media examples.

a. There are about 80 calories in one chocolate chip cookie. If we let *n* be the number of chocolate chip cookies eaten, write an algebraic expression for the number of calories consumed.

b. Brendan recently hired a contractor to do some necessary repair work. The contractor gave a quote of $450 for materials and supplies plus $38 an hour for labor. Write an algebraic expression to represent the total cost for the repairs if the contractor works for *h* hours.

c. A concession stand charges $3.50 for a slice of pizza and $1.50 for a soda. Write an algebraic expression to represent the total cost for *P* slices of pizza and *S* sodas.

Section 1.2: The Story of "x"

Example 1: Tell the story of x in each of the following expressions.

a. $x - 5$

b. $5 - x$

c. $2x$

d. x^2

Example 2: Tell the story of x in each of the following expressions.

a. $2x + 4$

b. $2(x + 4)$

c. $5(x - 3)^2 - 2$

Example 3: Write an algebraic expression that summarizes the stories below.

a. Step 1: Add 3 to x
Step 2: Divide by 2

b. Step 1: Divide x by 2
Step 2: Add 3

Example 4: Write an algebraic expression that summarizes the story below.

Step 1: Subtract x from 7
Step 2: Raise to the third power
Step 3: Multiply by 3
Step 4: Add 1

Section 1.2 – You Try

Complete the following problems.

a. Tell the story of x in the expression $\frac{x-3}{5}$

b. Write an algebraic expression that summarizes the story below:

Step 1: Multiply x by 2
Step 2: Add 5
Step 3: Raise to the second power.

Section 1.3: Evaluating Algebraic Expressions

Example 1: Find the value of each expression when $w = 2$. Simplify your answers.

$w - 6$ $6 - w$ $5w - 3$

w^3 $3w^2$ $(3w)^2$

$\frac{4}{5w}$ $\frac{5w}{4}$ 3^w

Example 2: Evaluate $ab + c$ given $a = -5$, $b = 7$, and $c = -3$

Example 3: Evaluate $a^2 - b^2$ given $a = -5$ and $b = -3$

Example 4: A local window washing company charges $11.92 for each window plus a reservation fee of $7.

a. Write an algebraic expression to represent the total cost from the window washing company for washing w windows.

b. Use this expression to determine the total cost for washing 17 windows.

Section 1.3 – You Try

Evaluate $b^2 - 4ac$ given $a = 5$, $b = -1$, $c = 2$. Show all steps as in the media examples.

Section 1.4: Applications

Example 1: The maximum heart rate is the highest heart rate achieved during maximal exercise. In general, you get the most benefits and reduce the risks when you exercise within your *target* heart rate zone. Usually this is when your exercise heart rate (pulse) is about 80 percent of your maximum heart rate. The formula $M = 0.8(220 - A)$, gives the *recommended* maximum heart rate, M, in beats per minute, for a person who is A years of age. What is the recommended maximum heart rate for a person who is 40 years old?

Example 2: A golfer strikes a golf ball. The height, H (in feet), of the ball above the ground after t seconds is given by the equation $H = -16t^2 + 80t$. Determine the height of the ball after 3 seconds. Show all of your work, and write your answer in a complete sentence.

Example 3: Simple interest is given by the formula $A = P + Prt$. Where A is the accrued value of the investment after t years, and P is the starting principal invested at an annual percentage rate of r, expressed as a decimal. Sally buys a $1,000 savings bond that pays 4% simple interest each year. How much will the bond be worth after 5 years?

Example 4: The formula $P = 266(1.009)^t$ estimates the population of the United States (in millions of people), t years after 1995.

a. Use this formula to estimate the U.S. population in 1995. Round your answer to the nearest million.

b. Use this formula to estimate the U.S. population in 2016. Round your answer to the nearest million.

Section 1.4 – You Try

Paul is planning to sell bottled water at the local carnival He buys 2 crates of water (2000 bottles) for $360 and plans on selling the bottles for $1.50 each. Paul's profit, P in dollars, from selling b bottles of water is given by the formula $P = 1.5b - 360$. Determine Paul's profit if he sells all 2000 bottles of water. Show all of your work, and write your answer in a complete sentence.

Section 1.5: Geometric Formulas

Example 1: The circumference of a circle with radius *r* is given by the formula $C = 2\pi r$

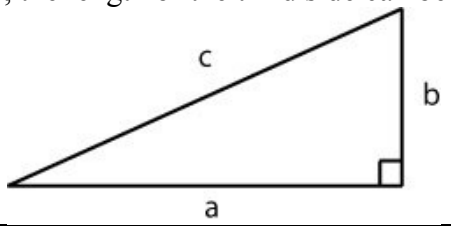
Determine the circumference of a circle with radius 32 cm. Write your answer in **exact form** (in terms of π) *and* in **approximate form**, rounded to the nearest hundredth.

Example 2: The formula for the volume of a cone of base radius *r* and height *h* is

$$V = \frac{1}{3}\pi r^2 h$$

Determine the volume of a cone with base radius 5 inches and height 12 inches. Write your answer in **exact form** (in terms of π) *and* in **approximate form**, rounded to the nearest hundredth.

The Pythagorean Theorem
The Pythagorean Theorem states that given any right triangle with legs *a* and *b*, and hypotenuse *c* as below, the following relationship is always true: $a^2 + b^2 = c^2$. Consequently, if the lengths of two sides are known, the length of the third side can be found using the formulas below:

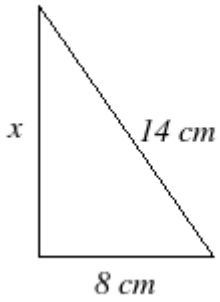


$$a = \sqrt{c^2 - b^2}$$

$$b = \sqrt{c^2 - a^2}$$

$$c = \sqrt{a^2 + b^2}$$

Example 3: Find the length of the leg *x* of the right triangle shown below. Write your answer in **exact form** *and* in **approximate form**, rounded to the nearest thousandth.

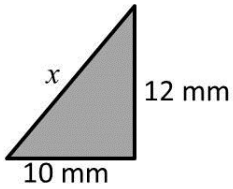


Section 1.5 – You Try

Complete the following problems. Show all steps as in the media examples.

a. The formula for the volume, *V*, of a cylinder of radius *r* and height *h* is $V = \pi r^2 h$. Determine the volume of a cylinder with radius 4 inches and height 10 inches. Write your answer in **exact form** (in terms of π) *and* in **approximate form**, rounded to the nearest hundredth. Include appropriate units in your answer.

b. Use the Pythagorean Theorem to find the length of side *x* of the right triangle shown below. Write your answer in **exact form** *and* in **approximate form**, rounded to the nearest hundredth. Include appropriate units in your answer.



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Unit 1: Practice Problems

Skills Practice

1. Tell the story of x in each of the following expressions.

a. $x - 11$ b. $x + 5$ c. $5x$

d. x^5 e. x^3 f. $2 - x$

g. $2x - 3$ h. $8x^2$ i. $(2x)^2$

j. $7 - 2x$ k. $5(7 - x)^3$ l. $\left(\frac{3x-8}{5}\right)^3$

2. Write an algebraic expression that summarizes the stories below.

a. Step 1: Add 8 to x
Step 2: Raise to the third power

b. Step 1: Divide x by 8
Step 2: Subtract 5

c. Step 1: Subtract 3 from x
Step 2: Multiply by 7

d. Step 1: Multiply x by 10
Step 2: Raise to the 3rd power
Step 3: Multiply by 2

e. Step 1: Add 5 to x
Step 2: Divide by 2
Step 3: Raise to the second power
Step 4: Add 8

f. Step 1: Raise x to the second power
Step 2: Multiply by 5
Step 3: Subtract from 9

g. Step 1: Subtract x from 2
Step 2: Multiply by -8
Step 3: Raise to the third power
Step 4: Add 1
Step 5: Divide by 3

h. Step 1: Multiply x by -4
Step 2: Add 9
Step 3: Divide by 2
Step 4: Raise to the fifth power

3. Find the value of each expression when $b = -8$. Simplify your answers.

   a. $b - 11$ b. $b + 5$ c. $5b$

   d. b^2 e. b^3 f. $2 - b$

4. Evaluate each of the following given $q = 10$.

   a. $2q - 3$ b. $8q^2$ c. $(2q)^2$

   d. $\frac{4}{7q}$ e. $7 - 2q$ f. 2^q

5. Find the value of each expression when $c = \frac{2}{3}$. Write your answers as proper fractions or mixed numbers in simplest form.

   a. $c - 5$ b. $c + \frac{3}{5}$ c. $\frac{3}{5}c$

   d. c^2 e. c^3 f. $\frac{2}{c}$

6. Evaluate the following expressions for the given values. Simplify your answers.

a. $\frac{-b}{2a}$ for $a = 6,\ b = 4$

b. $\frac{4x-8}{5+x}$ for $x = 3$

c. $\frac{3}{5}ab$ for $a = 8, b = 1\frac{2}{3}$

d. $3x^2 + 2x - 1$ for $x = -1$

e. $x^2 - y^2$ for $x = -3, y = -2$

f. $2x - 7y$ for $x = 5, y = 3$

g. $\sqrt{c^2 - a^2}$ for $a = 3, c = 5$

h. $\sqrt{b^2 - 4ac}$ for $a = -1, b = -5, c = 6$

Applications

7. Shea bought C candy bars for $1.50 each.
   a. Write an algebraic expression for the total amount Shea spent.

   b. Use this expression to determine the amount Shea will spend for 3 candy bars. Show all of your work and write your answer in a complete sentence.

8. Suppose sales tax in your town is currently 9%.
   a. Write an algebraic expression representing the sales tax for an item that costs D dollars.

   b. Use this expression to determine the sales tax for an item that costs $354. Show all of your work and write your answer in a complete sentence.

9. Ben bought M movie tickets for $8.50 each and B bags of popcorn for $3.50 each.
   a. Write an algebraic expression for the total amount Ben spent.

   b. Use this expression to determine the amount Ben will spend if he buys 6 movie tickets and 4 bags of popcorn. Show all of your work and write your answer in a complete sentence.

10. Noelle is 5 inches shorter than Amy. Amy is *A* inches tall.
    a. Write an algebraic expression for Noelle's height.

    b. Use this expression to determine Noelle's height if Amy is 5 feet 8 inches tall. Show all of your work and write your answer in a complete sentence.

11. Jamaal studied H hours for a big test. Karla studied one fourth as long.
    a. Write an algebraic expression for the length of time that Karla studied.

    b. Use this expression to determine the length of time that Karla studied if Jamaal studied for 5 hours and 20 minutes. Show all of your work and write your answer in a complete sentence.

12. A caterer charges a delivery fee of $45 plus $6.50 per guest.
    a. Write an algebraic expression to represent the total catering cost if G guests attend the reception.

    b. Use this expression to determine the total catering cost for if 80 people attend the reception. Show all of your work and write your answer in a complete sentence.

13. Tickets to the museum cost $18 for adults and $12.50 for children.
    a. Write an algebraic expression to represent the cost for A adults and C children to visit the museum.

    b. Use this expression to determine the cost for 4 adults and 6 children to attend the museum. Show all of your work and write your answer in a complete sentence.

14. Irena invested money into two mutual funds. During the first year, Fund A earned 2% interest and Fund B earned 3% interest.
    a. Write an algebraic expression for the total amount of interest Irena earned from both accounts during the first year.

    b. Use this expression to determine the total amount of interest earned during the first year if she invested $4,500 in Fund A and $1,200 in Fund B.

15. Ivan invested money into two mutual funds. Fund A earned 8% profit during the first year while Fund B suffered a 3% loss.
    a. Write an algebraic expression for the total profit Ivan earned from both mutual funds during the first year.

    b. Use this expression to determine the total amount of profit earned during the first year if he invested $3,800 in Fund A and $2,400 in Fund B.

16. The formula to convert from Fahrenheit to Celsius is $C = \frac{5}{9}(F - 32)$. The temperature on a summer day in Phoenix, Arizona is 115ºF. What would this temperature be in degrees Celsius? Round your answer to the nearest tenth of a degree. Show all work, and write your answer in a complete sentence.

17. Isabel has a headache, and takes 500mg of Tylenol. The amount, A, of Tylenol (measured in mg) remaining in her body after *n* hours is given by the formula $A = 500(0.882)^n$. How much of the Tylenol remains in her body after 4 hours? Show all work, and round your answer to the nearest hundredth. Write your answer in a complete sentence.

18. A person's Body Mass Index (BMI) is given by the formula $BMI = \frac{703W}{H^2}$, where W is the weight of the person in pounds, and H is the person's height, measured in inches. If a person is 5 feet 7 inches tall, and weighs 142 pounds, what is that person's BMI? Show all of your work. Round your answer to the nearest tenth. Write your answer in a complete sentence.

19. The formula for the volume, V, of a cylinder of radius r and height h is $V = \pi r^2 h$. Determine the volume of a cylinder with radius 3 inches and height 8 inches. Write your answer in exact form (in terms of π) and in approximate form, rounded to the nearest hundredth. Include appropriate units in your answer

20. The formula $A = \frac{1}{2}bh$ gives the area of a triangle with base b and height h. Determine the area of a triangle with base 4cm and height $2\frac{2}{3}$ cm. Write your answer as a proper fraction or mixed number in simplest form. Include appropriate units in your answer.

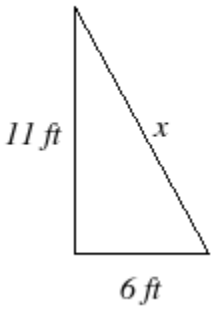
21. The formula $V = 9.54 + 0.08m$ represents the value of an investment (in thousands of dollars) after m months. Determine the value of this investment after two years.

22. The formula $E = 3861 - 77.2t$ gives the surface elevation (in feet above sea level) of Lake Powell t years after 1999. Use this formula to predict the surface elevation of lake Powell in the year 2016.

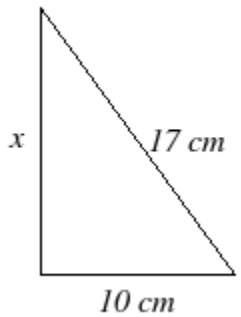
23. Simple interest is given by the formula $A = P + Prt$. Where A is the accrued value of the investment after t years, and P is the starting principal invested at an annual percentage rate of r, expressed as a decimal. Sally buys a $5,000 savings bond that pays 2.3% simple interest each year. How much will the bond be worth after 5 years?

24. The formula for compound interest is $A = P(1 + r)^t$ where A is the accrued amount after t years, P is the starting principal, and r is the annual interest rate expressed as a decimal. If you invest \$12,000 at an annual interest rate of 1.7% and leave it there for 30 years, what would your ending balance be? Round your answer to the nearest cent.

25. Use the Pythagorean Theorem to find the length of side x of the right triangle shown below. Write your answer in **exact form** and in **approximate form**, rounded to the nearest thousandth. Include appropriate units in your answer.



26. Use the Pythagorean Theorem to find the length of side x of the right triangle shown below. Write your answer in **exact form** and in **approximate form**, rounded to the nearest thousandth. Include appropriate units in your answer.



Extension

27. Evaluate $\frac{-b+\sqrt{b^2-4ac}}{2a}$ for a = 1, b = -4, and c = -3. Write your answer in **exact form** (simplified completely) and in **approximate form**, rounded to the nearest thousandth.

28. A pebble is dropped into a calm pond, causing ripples in the shape of concentric circles to expand on the surface of the water. The area of the outer ripple is given by the formula $A = \pi r^2$, where r is the radius of the outer ripple measured in inches. The formula $r = 3t$ gives the radius of the outer ripple after t seconds. Determine the area of the outer ripple after 5 seconds. Write your answer in exact form (in terms of π) and in approximate form, rounded to the nearest hundredth. Include appropriate units in your answer.

29. Given that $2x + 1 = 5$, evaluate each of the following expressions. Simplify your answers.

a. $8(2x + 1)$ b. $\frac{1}{2x+1}$ c. $(2x + 1)^2$ d. $(2x + 1) + 6$

e. $(2x + 1) – 8$ f. $2x + 5$ g. $2x – 1$ h. $2x$ i. $6x + 3$

30. The formula when interest is compounded *n* times per year is $A = P\left(1 + \frac{r}{n}\right)^{nt}$ where *A* is the accrued amount after *t* years, *P* is the starting principal, and *r* is the interest rate, expressed as a decimal, that is compounded *n* times per year. If you invest $1000 at an interest rate of 7%, and leave it there for 30 years, determine your ending balance if the interest is compounded

a. Once each year

b. Twice each year

c. Monthly

d. Daily

e. Explain what happens to the ending balance as the number of compoundings increases. Why does this occur?

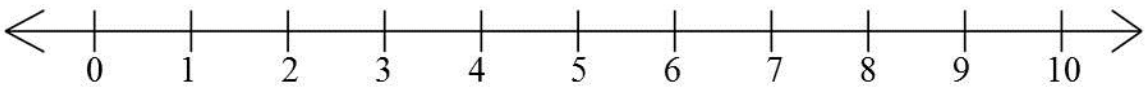
31. Working with square roots.

a. Without using your calculator, fill in the blanks below.

$\sqrt{1}$ =____	$\sqrt{\_\_}$ =5	$\sqrt{\_\_}$ =9
$\sqrt{4}$ =____	$\sqrt{\_\_}$ =6	$\sqrt{100}$ =____
$\sqrt{9}$ =____	$\sqrt{\_\_}$ =7	$\sqrt{\_\_}$ =11
$\sqrt{16}$ =____	$\sqrt{\_\_}$ =8	$\sqrt{144}$ =____

b. Without using your calculator, place each of the following on the number line below.

$\sqrt{2}$ $\sqrt{11}$ $\sqrt{40}$ $\sqrt{60}$ $\sqrt{99}$



c. Now use your calculator to evaluate each of the following. Round your answers to the nearest hundredth.

$\sqrt{2}$ = _____ $\sqrt{11}$ = _____ $\sqrt{40}$ = _____ $\sqrt{60}$ = _____ $\sqrt{99}$ = _____

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Unit 1: Review

1. A towing company charges $3.50 for each mile plus a nonrefundable reservation fee of $12. Determine an algebraic expression to represent the total cost for towing your car *m* miles.

2. Tell the story of *x* in the following expression $2(3 - x)^5$

3. Evaluate the following expressions for the given values. Show all of your work.

   a. $4x^2 - x + 3 \ \ for\ x = -5$

   b. $x^2 - y^2 \ \ for\ x = -5,\ y = -3$

4. You decide to leave an 18% tip for dinner.
   a. Write an algebraic expression to represent the tip if the bill for dinner was D dollars.

   b. Use this expression to determine the tip if the bill for dinner is $87.45. Round your answer to the nearest cent. Show your work.

5. The formula to convert from Fahrenheit to Celsius is $C = \frac{5}{9}(F - 32)$. The temperature on a summer day in Phoenix, Arizona is 113ºF. What would this temperature be in degrees Celsius? Show all work, and write your answer in a complete sentence.

6. The formula for the volume, V, of a cylinder of radius r and height h is $V = \pi r^2 h$. Determine the volume of a cylinder with radius 5 cm and height 40 cm. Give the exact answer (with π) and the approximate answer, rounded to the nearest hundredth. Include appropriate units in your answer.

7. The formula for compound interest is $A = P(1 + r)^t$ where A is the accrued amount after t years, P is the starting principal, and r is the annual interest rate expressed as a decimal. Bianca invests $5000 at an annual interest rate of 4% and leaves it there for 10 years. What will her ending balance be? Show all of your work. Round your answer to the nearest cent.

8. The formula $P = 289(1.009)^t$ estimates the population of the United States (in millions of people), t years after 2002. Use this formula to estimate the U.S. population in 2013. Show all of your work. Round your answer to the nearest million.