LESSON 10 – GEOMETRY I: PERIMETER & AREA

INTRODUCTION

Geometry is the study of shapes and space. In this lesson, we will focus on shapes and measures of one-dimension and two-dimensions. In the next lesson, we will work with shapes in three-dimensions.

The table below shows the specific objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Lesson Objective	Related Examples
Compute *perimeter* of different shapes	1, 2, YT5
Compute *circumference* of a circle	3, YT5
Explain the difference between *rounded form* and *exact form*	WE4
Compute distance around closed objects of unusual shape	6
Solve applications involving *perimeter* or *circumference*	7
Compute *area* of different shapes	8, 9, 10, YT11
Compute area of nonstandard shapes	12
Solve applications involving *area* and/or *perimeter*	13

KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- Perimeter
- Closed Triangle
- Geometric Shape/Open Geometric Shape
- Rectangle
- Square
- Circumference
- Circle
- Exact Form
- Rounded Form
- Π
- Diameter
- Radius
- Unit Square
- Area
- Height of a triangle

LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

MINILESSON

PERIMETER

Perimeter is a one-dimensional measurement that is taken around the outside of a closed geometric shape. Let's start our discussion of the concept of perimeter with an example.

Joseph does not own a car so must ride the bus or walk everywhere he goes. On Mondays, he must get to school, to work, and back home again. His route is pictured below.
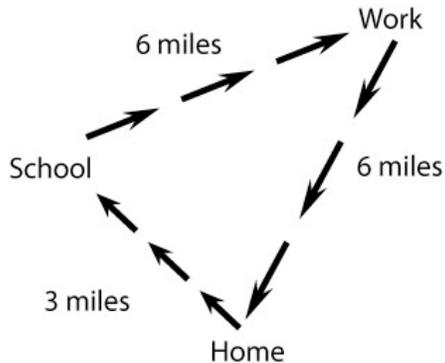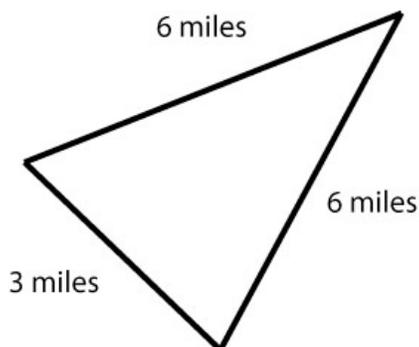

The obvious question to ask in this situation is, "how many miles does Joseph travel on Mondays"?

To compute, we each distance:

$$3 + 6 + 6 = 15$$

Joseph travels 15 miles on Mondays.

Another way to work with this situation is to draw a shape that represents Joseph's travel route and is labeled with the distance from one spot to another.

Notice that the shape made by Joseph's route is that of a closed geometric figure with three sides (a triangle). What we can ask about this shape is, "what is the *perimeter* of the triangle"?

Perimeter means "distance around a closed figure or shape" and to compute we add each length:

$$3 + 6 + 6 = 15$$

Our conclusion is the same as above. Joseph travels 15 miles on Mondays. However, what we did was model the situation with a geometric shape and then apply a specific geometric concept (*perimeter*) to computer how far Joseph traveled.

Notes on *Perimeter*:
- *Perimeter* is a one-dimensional measurement that represents the distance around a closed geometric figure or shape (no gaps).
- To find *perimeter*, add the lengths of each side of the shape.
- If there are units, include units in your final result. Units will always be of single dimension (i.e. feet, inches, yards, centimeters, etc…)

To compute perimeter, our shapes must be closed. The images below show the difference between a *closed figure* and an *open figure*.
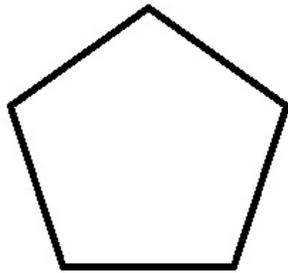
Closed figure:
There is an inside and an outside to the shape. To get from inside to outside, you must cross the boundary of the shape
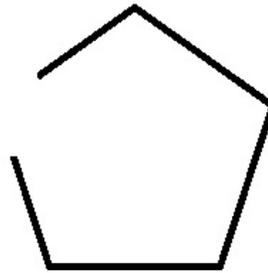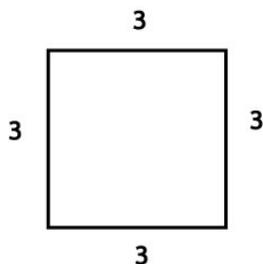
Open figure Not closed:
There isn't an inside or outside. Even a portion that seems enclosed can be reached without crossing the boundary of the shape.
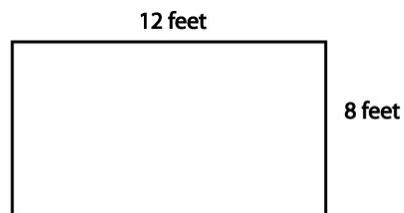
Example 1: Find the perimeter for each of the shapes below.
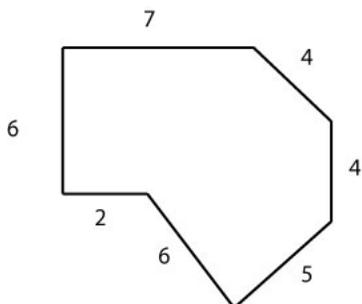
a. Add the lengths of each side.

b. Sometimes you have to make assumptions if lengths are not labeled.
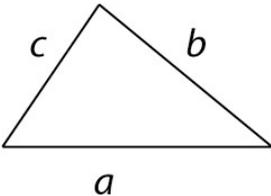
3

3   3

3

12 feet

8 feet

Example 2: How do we find the perimeter of this more complicated shape? Just keep adding those side lengths.

If you look closely at the shapes in the previous examples, you might notice some ways to write each perimeter as a more explicit formula. See if the results from what we have done so far match the formulas below.

Shape	Perimeter
Triangle with side lengths *a, b, c*	$P = a + b + c$
Square with side length *a*	$P = a + a + a + a$ $P = 4a$
Rectangle with side lengths *a, b*	$P = a + b + a + b$ $P = a + a + b + b$ $P = 2a + 2b$
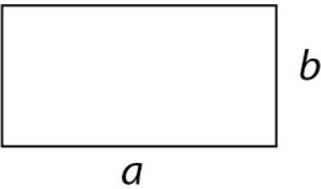
CIRCUMFERENCE

You may realize that we have not yet discussed the distance around a very important geometric shape: a circle! The distance around a circle has a special name called the *circumference*. To find the circumference of a circle, we use the formula below:

$$C = 2\pi r$$

In this formula, π is pronounced "pi", and is defined as the circumference of a circle divided by its diameter, $\pi = \dfrac{C}{d}$. We usually replace π with the approximation 3.14. The letter *r* represents the *radius* of the circle.

Let's see where the formula for circumference comes from. Below is a generic circle with radius r.
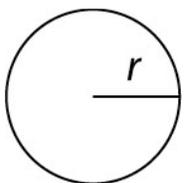
Notes about $C = 2\pi r$

Note: Remember that in the formula, when computing the circumference $C = 2\pi r$, we multiply as follows USUALLY substituting 3.14 in place of π

$$C = 2 \cdot 3.14 \cdot r$$

Often, the use of ( ) will help make the different parts of the formula easier to see:

$$C = (2) \cdot (3.14) \cdot (r)$$

Origins of $C = 2\pi r$

As mentioned earlier, the special number π is defined as the ratio of a circle's circumference to its diameter. We can write this in equation form as:

$$\frac{C}{d} = \pi$$

We know from our previous work that to identify the unknown, C, we can move *d* to the other side of the equation by writing:

$$C = \pi d$$

The diameter is all the way across the circle's middle so the diameter is twice the radius. We can update C in terms of the radius as:

$$C = \pi(2r)$$

With a little final rearranging of the order our parts are written in, we can say that:

$$C = 2\pi r$$

Let's use the formula to find the circumference of a few circles.

Example 3: Find the circumference of each of the following circles. Leave your answers first in *exact form* and then in *rounded form* (to the hundredths place). [Note that when a *radius* is given, its value is centered above a *radius* segment. When a *diameter* is given, its value is centered above a *diameter* segment.]
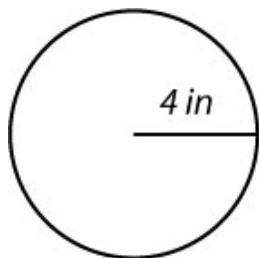
a.   b.
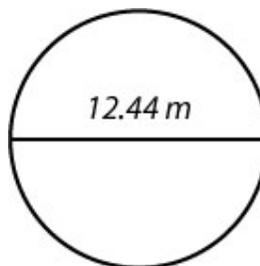
EXACT FORM VS. ROUNDED FORM

Exact Form vs. Rounded Form
• π is a number in exact form. It is not rounded.
• 3.14 is a rounded form approximation for π

Why does it matter which form we use? It matters because when we round, we introduce error into our final result. For this class, that error is usually acceptable. However, you will find in other subjects such as physics or chemistry, that level of accuracy is a concept of great importance. Let's see an example of the difference in forms.

Worked Example 4: The radius of the moon is about 1079 miles. What is the circumference?

Exact solution	Rounded solution
$C = 2\pi r = 2\pi(1079) = 2158\pi$	$C = 2\pi r = 2(3.14)(1079) \approx 6776.12$
To round FROM the exact solution , use the π button on your calculator to get	
$2158\pi \approx 6779.56$	

Notice that our final results are different. That difference is the error created by using 3.14 as an initial approximation for π. Read the directions carefully on each problem to see which form to use.

YOU TRY

5. Find the circumference or perimeter given in each described situation below. Include a drawing of the shape with the included information. Use the examples to help determine what shapes to draw. Show all work. As in the examples, if units are included then units should be present in your final result. Round to tenths unless indicated otherwise.

a. Find the perimeter of a square with side length 2.17 feet.

b. Find the perimeter of a rectangle with sides of length 4.2 and 3.8.

c. Find the perimeter of a triangle with sides of length 2, 5, 7.

d. Find the circumference of a circle with radius 6 inches. Present answer in exact form and also compute using 3.14 for π. Present rounded form to the nearest tenth.

e. Find the circumference of a circle with diameter 14.8 inches. Present answer in exact form and also compute using 3.14 for π. Present rounded form to the nearest tenth.

FINDING THE DISTANCE AROUND NON-STANDARD SHAPES

The basic formulas for perimeter of straight-line shapes and the circumference of a circle will help us find the distance around more complicated figures as in the example below.

**Example 6:** Find the distance around the following shape. Round final answer to tenths as use 3.14 for π.
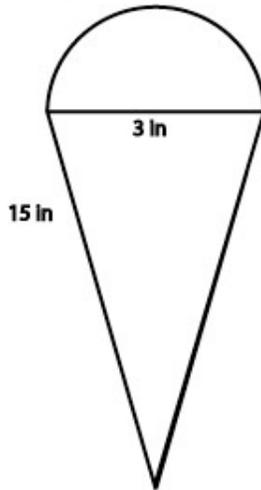
APPLICATIONS OF PERIMETER/CIRCUMFERENCE

Our knowledge of basic geometric shapes can be applied to solve problems like the next example.

Example 7: Wally wants to add a fence to the back of his house to make some room for his children to play safely (see diagram below). He began measuring his yard but got distracted and forgot to finish measuring before he went to the store. If he remembers that the back wall of his house is 15 yards long, does he have enough information to buy the fencing he needs? If so, how many feet should he buy?
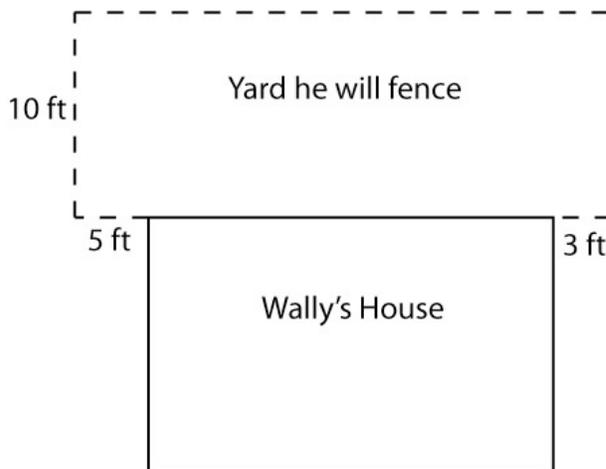
AREA

Let's take another look at Wally's backyard from Example 7 in order to introduce the next concept, *area*.

Wally successfully fenced his yard but now wants to add some landscaping and create a grassy area as shown below.
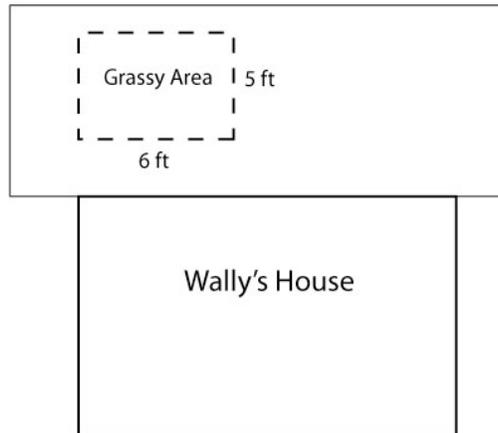
He heads down to the local lawn store and finds out that in order to determine how much sod he needs, he must figure out the square footage of the area he wants to add grass to. On his way home, he realizes that if he divides the grassy area into sections that are 1 foot by 1 foot and then counts them, he can determine the square footage. Here is the information Wally drew up when he got home.

6 top squares, 5 side squares
Each square is
1 ft x 1 ft = 1 square foot

Total unit squares = 30

Grassy Area = 30 square feet

Wally correctly determined the area of the rectangular grassy section to be 30 square feet.

Notes on *Area*:
- *Area* is a two-dimensional measurement that represents the amount of space inside a two-dimensional shape.
- To find the *area*, count the number of unit squares inside the shape.
- If there are units, include units in your final result. Units will always be two-dimensional (i.e. square feet, square yards, square miles, etc…)
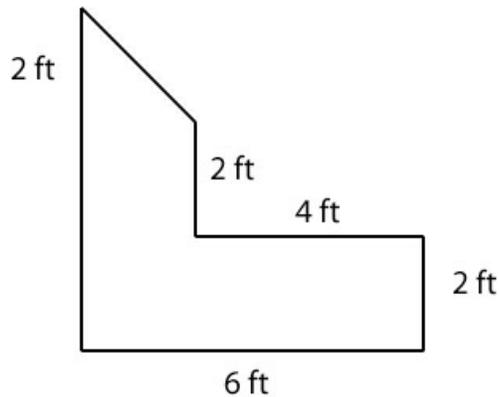
Example 8: Find the area for each of the shapes below.

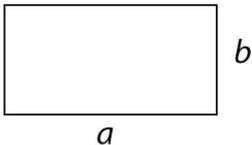a. Remember to count the unit squares inside the shape.

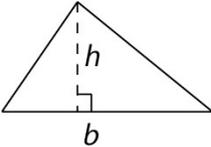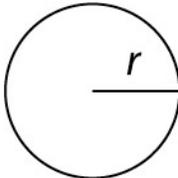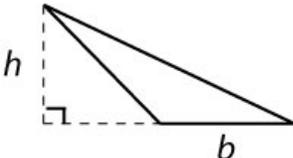b. Is there a pattern here that would make our work easier?
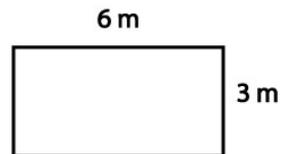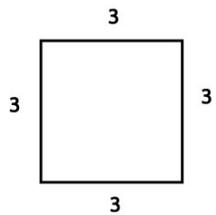
Example 9: How do we find the area for shapes that are more complicated? Break up the areas into shapes that we recognize and add the area values together.

If you look closely at the shapes in the previous examples, you might notice some ways to write each area as a more explicit formula. See if the results from what we have done so far match the formulas below.

Shape	Area
Square with side length a a	$A = a \cdot a$ $A = a^2$
Rectangle with side lengths a, b b a	$A = a \cdot b$ *(You will also see this as* $A = length \cdot width$*)*

The area formulas for the shapes below are more complicated to derive so the formulas are listed for you in the table.

Shape	Shape
Triangle with height h and base b h b $A = \dfrac{1}{2}bh = \dfrac{bh}{2}$ Read as "one-half base times height" Note that h is the straight-line distance from top of the triangle directly to the other side. The small box next to h indicates this. In math terms the box indicates a $90°$ (right) angle.	Circle with radius r r $A = \pi r^2$ Read as "*pi times radius squared*"
h b	If your triangle is as pictured at left, then the height is drawn and measured outside the triangle. The area formula is the same.

Example 10: Find the area for each described situation. Create a drawing of the shape with the included information. Show all work. As in the examples, if units are included then units should be present in your final result. Use 3.14 for π and round answers to tenths as needed.

a. Find the area of a rectangle whose length is 12.9 meters and height is one-third that amount.

b. Find the area of a triangle with base $24\frac{1}{2}$ inches and height 7 inches.

c. Find the area of a circle with radius $2\frac{1}{3}$ inches. Present answer in exact form and also compute rounded form using 3.14 for π. Present rounded form to the nearest tenth.

YOU TRY

11. Find the area given each described situation. Include a drawing of the shape with the included information. Show all work. As in the examples, if units are included then units should be present in your final result. Round answers to tenths unless otherwise indicated.

a. Find the area of a square with side length 4.2 feet.

b. Find the area of a rectangle with sides of length 4.2 and 3.8.

c. Find the area of a triangle with height 7 inches and base 12 inches.

d. Find the area of a circle with radius 6 inches. Present answer in exact form and also compute using 3.14 for π. Present rounded form to the nearest tenth.

FINDING THE AREA OF NON-STANDARD SHAPES

The basic formulas for area will help us find the area of more complicated figures as seen below. This is the same problem we found the perimeter for earlier.

Example 12: Find the area of the given shape. Compute using 3.14 for π and round to the nearest tenth.

3 In

14.92 In

15 In
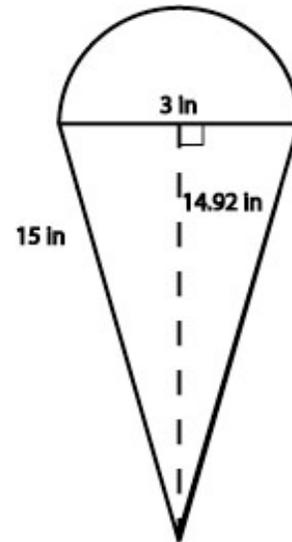
APPLICATIONS OF AREA/PERIMETER

We can combine our knowledge of area/perimeter to solve problems such as this one.

Example 13: Wally is still fixing up his house and has a flooring project to complete. He wants to buy enough bamboo flooring to cover the floor space in rooms A, C and hallway B and enough bamboo edging for baseboards in all the spaces as well. How many square feet of flooring and how many feet of baseboards should he buy?

3 ft   15 ft

B   A   10 ft

8 ft   C

8 ft

4 ft

Doorways
2 ft wide