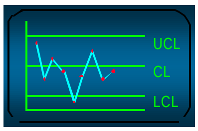
Chapter 14
Statistical Methods for Quality Control



Philosophies and Frameworks

- Quality
  - features and characteristics of a product or service that bears on its ability to satisfy given needs
- Total Quality
  - a people-focused management system aimed at continual increase in customer satisfaction at continually lower real cost
- Founding Fathers
  - Deming
    - advocated that managers develop a culture of commitment to quality
  - Juran
    - quality = fitness for use: quality planning, quality control, quality improvement
  - Common ground
    - management involvement
    - continuous improvement
    - importance of training
    - need to use Quality Control techniques

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Philosophies and Frameworks

- Baldrige National Quality Award (1987)
  - created to raise awareness and to recognize US organizations
- ISO 9000 (1987)
  - international standards defining what is needed to maintain an efficient quality conformance system
    - eg: measuring system calibration, record keeping systems
- Six Sigma (late 80's)
  - Motorola's goal: quality at rate of 3.4 dpmo
  - relies heavily on statistics
  - Quality Engineering
    - consider quality when designing products and processes and identify quality problems prior to production
  - Quality Control
    - inspections/measurements to determine if quality standards are being met

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Statistical Process Control

- Classical Hypothesis Testing generally involves making a one-time decision based on a single sample
- Statistical Process Control
  - many situations can change over time and must be monitored on an ongoing basis
  - why take frequent, periodic samples?

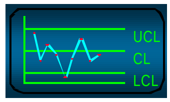
2 Sources of Process Variation

- Assignable Cause Variation
  - nonrandom influences arising out of factors that can be corrected
  - eg: wear & tear on machinery, operator error, contaminated materials
  - signals that the process is out of control
  - attempt to identify the underlying cause and take corrective action
  - how evidenced on control chart?
- Common Cause (Chance) Variation
  - random factors that cannot be avoided/corrected
  - are unavoidable even when the process is in control
  - eg: temperature changes, slight variations in materials
  - requires changing the process itself in order to reduce/eliminate
    - eg: invest in new machinery, better training, more supervision, new suppliers
- Driving analogy

Quality Control Inference

- H_o: The process is in control
  - i.e., no sources of assignable cause variation are present
- H_A: The process is out of control
  - i.e., sources of assignable cause variation are present
- Type I error?
- Type II error?

Control Charts



- Graphically depict process performance over time
- Constructed after a process has stabilized and is working as it is designed to
- Used to monitor variation in an important characteristic that affects quality of the product
- Control Chart has
  - Centerline
  - Upper Control Limit
  - Lower Control Limit
  - Data points for each sampled time period
    - each time period is a hypothesis test of the process being in control
- Aids in the detection of assignable cause variation
  - how?

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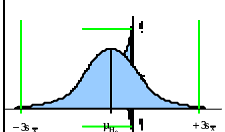
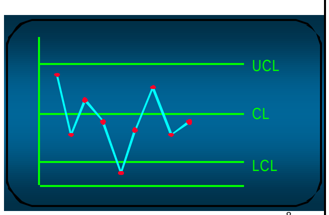
Control Charts

- Control limits are set at CL $\pm$ 3(standard errors)
  - by setting control limits at $\pm$ 3 std errors, α = ______
  - why?





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$\overline{x}$ Chart :Process Mean and Standard Deviation Known

- Tracks the value of the sample mean around the process' mean when the process is in control
- Centerline set at μ

$$\text{Control limits} = \mu \pm 3\sigma_{\overline{x}}$$

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$\bar{x}$ Chart: Process Mean and Standard Deviation **Un**known

- $\bar{x}$ vs. $\bar{\bar{x}}$
- Tracks the values of the sample mean around grand mean
- Centerline set at $\bar{\bar{x}}$
- Control Limits: $\text{UCL \& LCL} = \bar{\bar{x}} \pm A_2\bar{R}$
- Note: limits for $\bar{x}$ chart depend on sample ranges
- Acme Pipe assignment

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R-Chart

- Tracks process variation over time
- Higher variation suggests items tend to be dissimilar and of inconsistent quality
- Standard deviation is a more comprehensive measure, but:
  - range is easier to compute
  - range is more intuitive
  - not much information is lost when n is small (n<10)
- Centerline set at $\bar{R}$
- Control Limits: $UCL_R = \bar{R}D_4$ $LCL_R = \bar{R}D_3$
- Acme Pipe assignment
- Since $\bar{x}$ chart uses the average range, an R-chart is usually constructed first; only interpret/use $\bar{x}$ chart if R-chart indicates that process variation is in control

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Preliminary vs Revised Control Charts

- Any initial sample falling outside the preliminary control limits should be investigated
- If assignable cause variation is identified, those points should be dropped and the control limits should be recalculated using the remaining samples
- Use the revised limits for monitoring the process in the future

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Interpretation Of Control Charts

- A process is judged to be in control when:
  - all sample data points lie within UCL and LCL

  and
  - only random variation is present
    - even if all sample points fall within the three sigma limits, the process may not be in control
    - nonrandom patterns or trends can indicate a change in the process and a loss of control
    - examples of nonrandom patterns:
      - something other than 68% of points within ± 1 SD of centerline
      - something other than 95% of points within $\pm$ 2 SD of centerline
      - 9 points in a row on one side of a centerline
      - 6 points in a row that are consistently increasing or decreasing

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p Chart

- Applicable when a unit of product is judged as being either acceptable or not acceptable
- Tracks the proportion of defective items over time
- p vs. $\overline{p}$
- When p is unknown, pool all samples to estimate it
- Centerline set at p
- $\text{Control limits} = p \pm 3s_{\overline{p}} = p \pm 3\sqrt{\frac{p(1-p)}{n}}$
- if LCL < 0, set it at 0 since it is impossible to have fewer than 0% defectives
- MicroChip assignment

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Acceptance Sampling

- Sample items from a lot to infer whether the entire lot meets specifications
- Based on the number of defective items in the sample, either accept or reject the entire lot
- Advantages over 100% inspection:
  - less expensive
  - less product damage
  - fewer inspectors required
  - only option when testing is destructive
  - inspector fatigue (nib)

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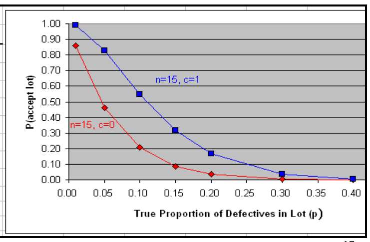
Acceptance Sampling

- H_0: ________________ vs H_A: ________________
- Type I error
  - Reject a good-quality lot
  - Producer's risk = P(reject lot | good-quality lot) = α
- Type II error
  - Accept a poor-quality lot
  - Consumer's risk = P(accept lot | poor-quality lot) = β
- Acceptance Criterion (c)
  - maximum number of defects allowed in sample without rejecting the lot
  - ensures that the AQL is maintained without rejecting more than a prescribed percentage of acceptable lots

Operating Characteristic Curve

- Illustrates how p, c, n affect the probability of accepting a lot

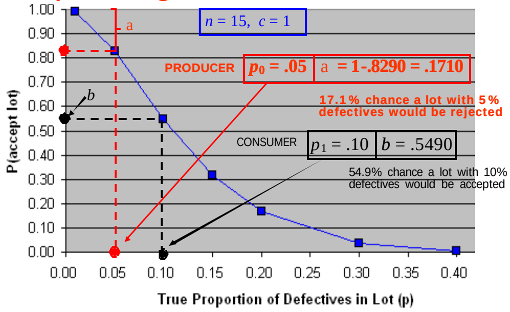
c	0	1
n	15	15
p		
0.01	0.8601	0.9904
0.05	0.4633	0.8290
0.10	0.2059	0.5490
0.15	0.0874	0.3186
0.20	0.0352	0.1671
0.30	0.0047	0.0353
0.40	0.0005	0.0052



Selecting an Acceptance Sampling Plan

- In formulating a plan, managers specify two values for the proportion of defectives in the lot
  - Producer's Risk
    - α = the probability that a lot with $\mathbf{p_0}$ defectives will be rejected
  - Consumer's Risk
    - β = the probability that a lot with $\mathbf{p_1}$ defectives will be accepted
- Then, the values of n and c are selected that result in an acceptance sampling plan that comes closest to meeting both the α and β requirements specified

Operating Characteristic Curve



Selecting an Acceptance Sampling Plan: A Special Case

- Special case: $p_0 = p_1$
- Determine c by using Cumulative Binomial tables
  - select column for $p_0 = p_1$
  - locate c as the cumulative probability exceeding (1-Producer's Risk)
  - if more than c defects are found in the sample, reject the lot

$$P_B(x \leq c \mid n, p) \geq (1 - \text{Producer's Risk})$$

Multi-Stage Sampling Designs

- Rather than using a single sample to determine whether to accept a lot
- Take a preliminary sample…. then, depending on its result either: (1) accept the lot, (2) reject the lot, or (3) take another sample, pooling it with previous samples, and then decide
- Is generally more efficient than a single-sampling design
  - i.e., a smaller total sample size to inspect/test