Chapter 9
Hypothesis Testing

Hypothesis Testing Concepts

- Hypothesis Testing a.k.a. Scientific Method
- Begin with an assumed value for a parameter... take a random sample... compare the sample result to the hypothesized value:
  - if they're similar, don't reject the hypothesis
  - if they're significantly different, revise or reject the hypothesis
- Reject the hypothesis only if the sample result is unlikely to occur due to sampling error when hypothesis is true

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Fizz Bottling Problem

- Bottles should average 12 ounces
- A sample of n=50 & finds mean of 12.03 ounces
- The standard and the observed sample mean are .03 ounces different
- the difference is due to either:
- 1)

or

- 2)

- Which is it reasonable to attribute the difference to?

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1) State the Hypotheses

H_0:________ **Null Hypothesis** (always gets equality) is believed true until sample evidence clearly indicates otherwise

H_A:________ **Alternative Hypothesis** indicates what is believed when H_0 is rejected

- Each hypothesis makes a statement regarding a parameter's value
- Table of consequences

action	H_0 true	H_0 false
not reject H_0	Correct decision	**Type II error**
reject H_0	**Type I error**	Correct decision

- What could cause a Type I or II error?

2) Level of Significance

- Denoted α
- Commonly used 1%, 5%, 10%
- α is the maximum probability of a Type I error the researcher is willing to risk
- Said earlier "reject H_0 when the sample result is unlikely to occur due to sampling error when H_0 is true." How unlikely...? α
- Could μ=12, yet X-bar=12.02? 12.08? 12.20? 12.25?
- α=.01, so reject only if the sample result has <1% chance of occurring due to sampling error when H_0 is true
  - i.e. <1% chance of getting a biased a sample that makes it look as though H_0 is false when it is actually true
- α = P(reject H_0 | H_0 true) = P(Type I error)
- α corresponds to the area(s) devoted to rejection

3) Specify the Test Statistic

- The point estimator of parameter being tested
- Here we're testing hypothesized value of μ so the test statistic is ___, whose sampling distribution follows ______ distribution.

6 Formal Steps of Hypothesis Testing

4) Critical Value

- a.k.a. Action Limit
- The value of sample test statistic that leads to rejection of H_0
- Depicted under the assumption that H_0 is true
- What sample result(s) would cause you to reject H_0?
- Decision rule:
  - Don't reject H_0 if ___________
  - Reject H_0 if ___________

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6 Formal Steps of Hypothesis Testing

5) Collect/Analyze Sample

- Compute the standard score for the sample result

$$Z = \frac{\overline{X} - \mu_{H_0}}{\sigma_{\overline{X}}}$$

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6 Formal Steps of Hypothesis Testing

6) Make a Statistical Decision

- Compare the location of the sample result with the critical values(s), then determine whether or not to reject H_o
- State your conclusion in terms of the decision scenario and refer to the hypothesized value of the parameter being tested.

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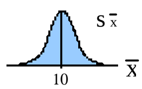
Forms of Null and Alternative Hypotheses

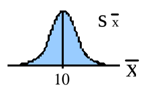
- $H_0: \mu \geq 10$
- $H_A: \mu < 10$

SDOM if Ho true



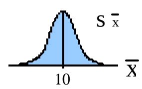
- $H_0: \mu \leq 10$
- $H_A: \mu > 10$

SDOM if Ho true



- $H_0: \mu = 10$
- $H_A: \mu \neq 10$

SDOM if Ho true



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Using x-bar to Estimate μ: σ "Known" (Review Chapter 8)

- x-bar is a random variable that is described by a probability distribution called the sampling distribution of the mean
- Sampling Distribution of the Mean (SDOM)
  - the distribution of possible sample means
  - follows z-distribution when:
    - you have historical data to estimate σ AND
    - either (any shaped population, provided $n \geq 30$)
    - or ($n \geq 15$ and symmetric population)
- Standard Error of the Mean
  - measures ROSE when use x-bar to estimate μ

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$

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Hypothesis Tests Involving μ: σ "Known"

- Use z to set critical value(s)
- Two-tailed test for μ
  - Fizz Bottling was an example
- One tailed tests for μ
  - same process, only one critical value, all of alpha in one tail
  - TurboRead assignment

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p-value Approach

- Measures the amount of support provided by the sample for the null hypothesis
- The probability of obtaining a sample result at least as extreme as the result obtained, given that H_O is true.
- Once the value of the test statistic is known, calculate its standard score and find the area in the tail beyond the sample result. This area is the p-value.
  - note: if performing a two-tailed test, double the tail area
- **if p-value $\le$ a, reject H_O, otherwise don't reject H_O**
- Many statistical analysis programs don't ask for your desired significance level (α). They report the p-value for you to compare to your desired significance level
- TurboRead assignment
- Fizz Bottling assignment

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Using x-bar to Estimate μ: σ **Un**Known (Review Chapter 8)

- When σ is unknown, we must use sample data to estimate it with s
- But then we're using the same sample for 2 purposes:
  - to calculate **x-bar** in order to estimate **μ**
  - to calculate **s** in order to estimate s
- Sampling Distribution of the Mean will follow a t-distribution when:
  - you lack historical data to estimate σ AND
  - either (Normal population, regardless of n)
  - or (symmetric population, provided n $\ge$ 15)
  - or (skewed population, provided n $\ge$ 50)

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

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Hypothesis Test of μ: σ Unknown

- Same 6-step process, use t_{n-1} instead of z
- OkayData assignment
- p-values are awkward for t-distribution since our t-tables have only selected areas
  - there are lots of gaps between the reported areas so can only estimate tail areas
  - OKAYData

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Relation Between Confidence Intervals and Hypothesis Tests

- A confidence interval is similar to a 2-tailed hypothesis test

$$\text{Chapter 8} \quad \text{CI for } \mu = \bar{x} \pm z\frac{s}{\sqrt{n}}$$

$$\text{Chapter 9} \quad \text{Critical Values} = \mu_{H_0} \pm z\frac{s}{\sqrt{n}}$$

- If the CI contains μ_{H0}, don't reject H_0
- If the CI does not contain μ_{H0}, can reject H_0

Review Chapter 8

Using p-bar to Estimate p

- Sampling Distribution of the Proportion (SDOP)
  - the distribution of sample proportions
  - follows the BINOmial distribution, but can use z when:
    - np $\geq$ 5 and
    - n(1-p) $\geq$ 5
- Standard Error of the Proportion
  - measures ROSE when use p-bar to estimate p
  - twist: the null will refer to a hypothesized value and it is to be believed until sample evidence indicates otherwise and serves as the basis for the standard error, not p-bar

$$\sigma_{\bar{p}} = \sqrt{\frac{p_{H_0}(1-p_{H_0})}{n}}\sqrt{\frac{N-n}{N-1}}$$

Tests About a Population Proportion

- Can be either 1-tailed or 2-tailed
- State University Parking assignment

Type I and Type II Errors

- Type I error
  - reject H_0 when it is true
  - α = P(Type I) = P(reject | H_0 true)
- TYPE II error
  - fail to reject H_0 when it is false

action	H_0 true	H_0 false
not reject H_0	Correct decision	**Type II error**
reject H_0	**Type I error**	Correct decision

- Example: American criminal justice system
- H_0:______________ Type I error ______________
- H_A:______________ Type II error ______________

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