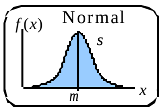
Chapter 6
Continuous Probability Distributions

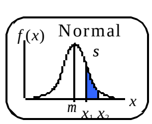


Continuous Probability Distributions

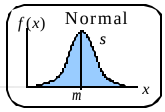
- Describes the possible outcomes and probabilities of occurrence for a continuous random variable
  - can't list all the possible values since there are an infinite number of values along the relevant portion of measurement scale
- Are defined by an equation called a probability density function (PDF)
  - f(X)= [some equation involving X]

Continuous Probability Distributions: Concepts

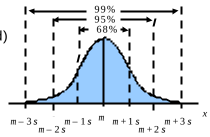
- The probability of the random variable assuming a value within the interval from x_1 to x_2 is equal to the relative area under the probability density function between x_1 and x_2
- Probability = % of area under PDF
- Total area beneath PDF = 1.0000
- Area beneath a single point = 0.0000
- Actually use calculus to find probabilities
- We'll use tables in book



Normal Distribution



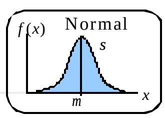
- A continuous probability distribution
- Properties
  - unimodal & symmetrical (i.e., bell-shaped)
  - mean = median = mode
  - ≈68% of observations within $\mu \pm 1\sigma$
  - ≈95% of observations within $\mu \pm 2\sigma$
  - ≈99% of observations within $\mu \pm 3\sigma$



- Prominent because
  - many random variables roughly follow this pattern of distribution
  - it can be used to approximate Binomial, Poisson and other distributions under certain conditions

4

Normal Distribution



- PDF
  - the height of the Normal curve at any point X once you supply the values for μ and σ

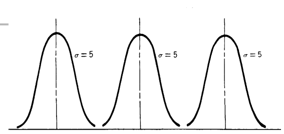
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

- where:
  - μ = mean
  - σ = standard deviation
  - π = 3.14159
  - e = 2.71828

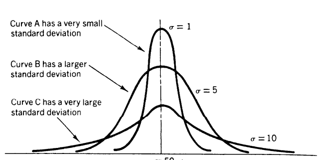
5

Normal Distribution

- The shape and location of any Normal distribution are determined by two parameters: μ and σ



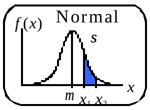
- There is a different Normal curve for each set of values for μ and σ



6

Standard Normal Distribution

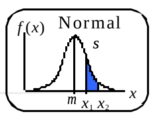
- Allows the use of table for determining areas beneath any normal distribution, regardless of μ and σ
- We'll denote as $P_N(x_1 \le x \le x_2 \mid \mu, \sigma)$
- Calculate the z-score for the endpoint(s) of the interval of interest, then use the z-score to look up the associated area(s) in the table



$$z = \frac{x - \mu}{\sigma}$$

7

Calculating Normal Probabilities



- You have an interval and need to determine the probability (area beneath the curve) contained within that interval
  - determine μ & σ (givens)
  - sketch a normal curve and label μ & σ
  - shade the interval of interest on the sketch
  - calculate the z-score for the interval's endpoints
  - look up the area(s) for the z score
  - determine the area beneath the curve for the interval
- Household Income

8

Calculating an X-value From a Known Probability

- You have a probability (area beneath the curve) and need to determine the endpoint(s) that define the interval
  1. determine μ & σ (givens)
  2. sketch a Normal curve and label μ & σ
  3. determine the given probability (area beneath the curve) for the interval of interest
  4. shade the appropriate interval on the sketch
  5. use the area to look up the appropriate z-score
  6. use the z score, μ, & σ to back-solve for x
- Household Income

9