

NOTE: All work must be shown including plugging into formulas.

1. [18 pts] Circle the item in the brackets that makes each statement true.
 - a) The number of cars in line at the emissions test center is an example of [**discrete** | **continuous**] data.
 - b) The length of time a particular vehicle spends waiting in line at the emissions test center is an example of [**nominal** | **ordinal** | **interval** | **ratio**] level data.
 - c) For a given data set, a stem-and-leaf display provides more information than a histogram. [**True** | **False**]
 - d) An ordered array can be constructed for all of the following *except* [**nominal** | **ordinal** | **interval** | **ratio**] level data.
 - e) Given: $P(A)=.20$, $P(B)=.40$ and $P(A|B)=0$, then events A and B are said to be [**mutually exclusive** | **complements** | **statistically independent** | **statistically dependent**]
 - f) Which of the following Z-scores (standard scores) is the most likely to be an *outlier*? [**-2.0** | **0** | **+4.5**]

2. [8 pts] A particular sample contained 77 items. The following frequency distribution has been partially completed.

Limits	Freque ncy
3-9	22
10-16	16
??-??	11
24-30	??

- a) What are the limits for the *third* class? _____
 - b) What is the frequency for the *last* class? _____
 - c) What is the relative frequency for the *first* class? _____
 - d) What is the *cumulative* frequency for the *third* class? Use the “less than” orientation. _____

3. [4 pts] The manager of the Scottsdale emissions testing station reviewed last week’s payroll records and found that her 4 workers were paid for the following number of hours at their respective wage rates. Determine the station’s average hourly wage rate.

Worker	Hours	Wage Rate
Adams	22	6.50
Baker	48	9.55
Charles	10	4.25
Zin	30	6.00

4. [4 pts] The Scottsdale station emission tested 12,460 vehicles during the 1988 calendar year. In the 1995 calendar year, it tested 27,412 vehicles. What is this station’s average annual rate of increase in vehicles tested?

5. [15 pts] A study was conducted to determine the amount of time cars spend at emissions test stations. A sample of 318 cars was gathered. The longest time was 1,403 seconds and the shortest was 110 seconds.
- A frequency distribution is to be prepared and the classes are to be of equal width. How many classes would you recommend?
 - What interval width would you recommend for each of the classes in part (a) above?

The frequency distribution was constructed and appears *bell-shaped*. The average time was 650 seconds with a standard deviation of 215 seconds. *Hint: make a sketch.*

- What proportion of cars require *more* than 220 seconds?
 - What is the standard score (z score) for a car that needed 1,000 seconds?
 - StatCrunch* reported that Q_3 had the value 746.2 seconds. Interpret this value.
6. [18 pts] The following values represent the number of cars processed at a particular emissions testing facility each hour for a random sample of 10 hours.

12 17 22 24 38 48 50 55 56 58

Determine each of the following:

a) $\Sigma X =$ _____

b) $\Sigma X^2 =$ _____

c) Median [_____]

d) Mean [_____]

e) Variance [_____]

f) 20th percentile [_____]

7. [4 pts] When should the *coefficient of variation* be used? How is it computed?

8. [14 pts] A sample of 250 cars was collected to determine whether older cars tend to fail the emissions test at a higher rate than newer cars. The following table has been constructed. *Carry all calculations to 3 decimal digits.*

	Passed	Failed	Total
Older car	46	34	80
Newer car	134	36	170
Total	180	70	250

- a) What is the probability a randomly selected car passed its emissions test? _____
- b) What is the probability a randomly selected car was a newer car? _____
- c) What is the probability a randomly selected car passed the emissions test or was a newer car or both?
- d) A guy just drove out in an old jalopy. What is the probability his car passed its emissions test? _____
- e) Based on the cars in this sample, does it appear as though older cars tend to fail the emissions test at a higher rate than newer cars?. Cite numerical proof.
9. [5 pts] An emissions test machine consists of 3 *independent* parts (A, B and C). The machine only works when each of these 3 parts work correctly. The probabilities of the parts working correctly are: $P(A)=.90$, $P(B)=.80$, $P(C)=.65$. What is the probability that the test machine is working?
10. [10 pts] When a vehicle fails an emissions test, its owner is required to have repairs made before the vehicle is retested. Surveys indicate that 80% of owners pay a professional mechanic to make repairs, while the remaining 20% of owners make the repairs themselves.

It is estimated that 95% of the vehicles repaired by a *professional mechanic* pass the retest inspection and that only 40% of *owner-repaired* vehicles pass the retest inspection.

One particular vehicle just passed its retest inspection. What is the probability that a professional mechanic made the repairs?

NOTE: All work must be shown including plugging into formulas.

1. [18 pts] Circle the item in the brackets that makes each statement true.
 - a) The number of cars in line at the emissions test center is an example of [discrete | continuous] data.
 - b) The length of time a particular vehicle spends waiting in line at the emissions test center is an example of [nominal | ordinal | interval | ratio] level data.
 - c) For a given data set, a stem-and-leaf display provides more information than a histogram. [True | False]
 - d) An ordered array can be constructed for all of the following *except* [nominal | ordinal | interval | ratio] level data.
 - e) Given: $P(A)=.20$, $P(B)=.40$ and $P(A|B)=0$, then events A and B are said to be [mutually exclusive | complements | statistically independent | statistically dependent]
 - f) Which of the following Z-scores (standard scores) is the most likely to be an outlier? [-2.0 | 0 | +4.5]

2. [6 pts] A particular sample contained 77 items. The following frequency distribution has been partially completed.

Limits	Frequency
3-9	22
10-16	16
17-23	11
24-30	17

- a) What are the limits for the *third* class? 17-23
- b) What is the frequency for the *last* class? 28 $(77 - 22 - 16 - 11)$
- c) What is the relative frequency for the *first* class? $\frac{22}{77} = .2857$
- d) What is the *cumulative* frequency for the *third* class? Use the "less than" orientation. 49

3. [4 pts] The manager of the Scottsdale emissions testing station reviewed last week's payroll records and found that her 4 workers were paid for the following number of hours at their respective wage rates. Determine the station's average hourly wage rate.

Worker	Hours	Wage Rate	$X \cdot W$
Adams	22	6.50	143.00
Baker	48	9.35	448.80
Charles	10	4.25	42.50
Zio	30	6.00	180.00
			<u>823.90</u>

$$M_w = \frac{823.90}{110} = \$7.49/\text{hr}$$

4. [4 pts] The Scottsdale station emission tested 12,460 vehicles during the 1988 calendar year. In the 1993 calendar year, it tested 27,412 vehicles. What is this station's average annual rate of increase in vehicles tested?

$$GM = \sqrt[93-88]{\frac{27412}{12460}} - 1 = .119290$$

5. [15 pts] A study was conducted to determine the amount of time cars spend at emissions test stations. A sample of 318 cars was gathered. The longest time was 1,403 seconds and the shortest was 110 seconds.

a) A frequency distribution is to be prepared and the classes are to be of equal width. How many classes would you recommend?

$$Z \approx 318 \quad Z_8 = 256 \quad Z_9 = 312 \quad \text{so } 9 \text{ classes.}$$

b) What interval width would you recommend for each of the classes in part (a) above?

$$\text{Min width} = \frac{1403 - 110}{9} = 143.66, \text{ round up to } 150 \text{ sec.}$$

The frequency distribution was constructed and appears bell-shaped. The average time was 650 seconds with a standard deviation of 215 seconds. Hint: make a sketch.

c) What proportion of cars require more than 220 seconds?

$$Z = \frac{220 - 650}{215} = -2.00$$



d) What is the standard score (z score) for a car that needed 1,000 seconds?

$$Z = \frac{1000 - 650}{215} = +1.62$$

EMPIRICAL RULE predicts 95% will be within ± 2 standard deviations so 5% left & 1/2000000 left so 97.5%

e) StatCrunch reported that Q_3 had the value 746.2 seconds. Interpret this value.

APPROX 75% OF CARS TOOK 746.2 SECONDS OR LESS TO BE TESTED.

6. [18 pts] The following values represent the number of cars processed at a particular emissions testing facility each hour for a random sample of 10 hours.

12 17 22 24 38 48 50 55 56 58
1 2 3 4 5 6 7 8 9 10

Determine each of the following:

a) $\Sigma X = 380$

$$b) \Sigma X^2 = 17,266$$

c) Median [43]

$$LDC = \frac{10+1}{2} = 5.5$$

$$MEO = \frac{38 + 48}{2} = 43$$

d) Mean [38]

$$\bar{X} = \frac{380}{10} = 38$$

e) Variance [314]

$$S^2 = \frac{\Sigma Y^2 - n\bar{X}^2}{n-1}$$

$$= \frac{17266 - 10(38)^2}{10-1} = 314$$

f) 20th percentile [20]

$$i = \frac{20(10)}{100} = 2$$

$$P_{20} = \frac{17 + 22}{2} = 19.5 \approx 20$$

7. [4 pts] When should the coefficient of variation be used? How is it computed?

WHEN YOU NEED TO COMPARE THE DISPERSION OF 2 DATA SETS HAVING DIFFERENT MAGNITUDES OR DIFFERENT UNITS OF MEASUREMENT.

$$CV = \frac{S}{\bar{X}} \times 100$$

8. [14 pts] A sample of 250 cars was collected to determine whether older cars tend to fail the emissions test at a higher rate than newer cars. The following table has been constructed. Carry all calculations to 3 decimal digits.

	Passed	Failed	Total
Older car	46	34	80
Newer car	134	36	170
Total	180	70	250

- a) What is the probability a randomly selected car passed its emissions test? $P(P) = \frac{180}{250} = 0.720$
- b) What is the probability a randomly selected car was a newer car? $P(N) = \frac{170}{250} = 0.680$
- c) What is the probability a randomly selected car passed the emissions test or was a newer car or both?
 $P(P \cup N) = P(P) + P(N) - P(P \cap N) = 0.72 + 0.68 - \frac{134}{250} = 0.864$
- d) A guy just drove out in an old jalopy. What is the probability his car passed its emissions test? $P(P|O) = \frac{P(P \cap O)}{P(O)} = \frac{46/250}{80/250} = 0.575$
- e) Based on the cars in this sample, does it appear as though older cars tend to fail the emissions test at a higher rate than newer cars? *Give numerical proof.*

$$P(F|O) = \frac{P(F \cap O)}{P(O)} = \frac{34/250}{80/250} = 0.425$$

$$P(F|N) = \frac{P(F \cap N)}{P(N)} = \frac{36/250}{170/250} = 0.212$$

} Yes, old cars fail at a higher rate than new cars.

9. [5 pts] An emissions test machine consists of 3 independent parts (A, B and C). The machine only works when each of these 3 parts work correctly. The probabilities of the parts working correctly are: $P(A) = .90$, $P(B) = .80$, $P(C) = .63$. What is the probability that the test machine is working?

$$P(\text{working}) = P(A \cap B \cap C) = P(A) \times P(B) \times P(C) = 0.9 \times 0.8 \times 0.63 = 0.4536$$

10. [10 pts] When a vehicle fails an emissions test, its owner is required to have repairs made before the vehicle is retested. Surveys indicate that 80% of owners pay a professional mechanic to make repairs, while the remaining 20% of owners make the repairs themselves.

It is estimated that 95% of the vehicles repaired by a professional mechanic pass the retest inspection and that only 40% of owner-repaired vehicles pass the retest inspection.

One particular vehicle just passed its retest inspection. What is the probability that a professional mechanic made the repairs?

Given: $P(\text{PRO}) = 0.80$ $P(\text{SELF}) = 0.20$

$P(\text{PASS}|\text{PRO}) = 0.95$ $P(\text{PASS}|\text{SELF}) = 0.40$

Find: $P(\text{PRO}|\text{PASS}) = \frac{P(\text{PRO} \cap \text{PASS})}{P(\text{PASS})} = \frac{0.76}{0.76 + 0.08} = \frac{0.76}{0.84} = 0.9048$

	1st	2nd	JOINT	PROB
80 PRO	0.95 PASS	0.05 FAIL	PRO \cap PASS	0.76
			PRO \cap FAIL	0.04
20 SELF	0.40 PASS	0.60 FAIL	SELF \cap PASS	0.08
			SELF \cap FAIL	0.12
				1.00