

Chapter 11 Comparisons Involving Proportions

use $(\bar{p}_1 - \bar{p}_2)$ to estimate $(p_1 - p_2)$

Hypothesis Tests for $p_1 - p_2$

- $(p_1 - p_2)$ is estimated by $(\bar{p}_1 - \bar{p}_2)$
- How could we compare the proportion of honor students at SCC vs. ASU?
- Common to test that there is **no difference** between population proportions
 - $H_0: p_{\text{SCC}} - p_{\text{ASU}} = 0$ vs. $H_A: p_{\text{SCC}} - p_{\text{ASU}} \neq 0$
- Can test any hypothesized difference
 - eg: the proportion of honors students at SCC is at least 15% points higher than at ASU
 - $H_0: p_{\text{SCC}} - p_{\text{ASU}} \geq .15$ vs. $H_A: p_{\text{SCC}} - p_{\text{ASU}} < .15$

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Hypothesis Tests for $p_1 - p_2$

- Follow the same 6-step process
- Sampling Distribution of the Difference Between Proportions
 - describes possible values for $(\bar{p}_1 - \bar{p}_2)$
 - follows the BINomial distribution but can use z when:
 - 1) $np \geq 5$ for each sample AND
 - 2) $n(1-p) \geq 5$ for each sample

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Hypothesis Tests for $p_1 - p_2$

- Standard Error of the Difference Between Proportions
 - measures the ROSE when testing the difference between population proportions with difference between sample proportions

$$\sigma_{\bar{p}_1 - \bar{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

- Estimated Std Error of Difference Between Proportions

$$s_{\bar{p}_1 - \bar{p}_2} = \sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$$

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Special Case: Hypothesis Test for $p_1 = p_2$

- When testing that there is **no difference** between population proportions ($H_0: p_1 - p_2 = 0$), the standard error is based on a **pooled estimate of the common proportion**:

$$\hat{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2}$$

- Estimated Std Error of Difference Between Proportions

$$s_{\bar{p}_1 - \bar{p}_2} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

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Hypothesis Tests for $p_1 - p_2$

- Test statistic is $(\bar{p}_1 - \bar{p}_2)$

$$z = \frac{(\bar{p}_1 - \bar{p}_2) - (p_1 - p_2)}{s_{\bar{p}_1 - \bar{p}_2}}$$

- Textbooks America
 - Connection: Contingency Table

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