Chapter 5
Discrete Probability Distributions

Random Variable

- A process which
  - has more than one possible outcome
  - you don't know in advance which outcome will occur
- 2 major types (Ch 1)
  - continuous
    - can take on any value along relevant portion of measurement scale
    - can't list every possible outcome
    - egs: height, weight, age
  - discrete
    - can take on one of a finite number of values along the relevant portion of a measurement scale
    - can list every possible outcome
    - egs: # absent, # defective, # sales
    - Hoover assignment

Probability Distributions

- Describe the possible outcomes of a random variable & their associated probabilities of occurrence

2 major types

- Continuous (Chs 6 & 8)
  - Normal, t
- Discrete (Ch 5)
  - Uniform, Binomial, Poisson

Discrete Probability Distributions

- Can take on one of a finite number of values along the relevant portion of a measurement scale
  - can list every possible outcome
  - Hoover example
- Probability Mass Function (nib)
  - the function that defines a discrete probability distribution
  - can be plotted
- Required conditions for a discrete probability function:

$f(x) \geq 0$ $\Sigma f(x) = 1$

4

Discrete Uniform Probability Distribution

- The simplest discrete probability distribution
- Applies when the possible values of the random variable are all equally likely to occur
  - eg: a fair die

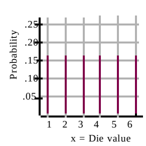
- PMF

$p(x) = 1/n$

where:

n = the number of values the random variable may assume

x	p(x)
0	.1667
1	.1667
2	.1667
3	.1667
4	.1667
5	.1667
6	.1667
	1.0000



5

Summarizing Discrete Probability Distributions

- Skew
- Expected Value
  - weighted mean, long run average value of the random variable

$$\mu = E(x) = \Sigma[x \ p(x)]$$

- Variance
  - dispersion among the possible values of the random variable

$$\sigma^2 = \Sigma(x - \mu)^2 P(x) = \Sigma[x^2 P(x)] - \mu^2$$

Computational formula

- Hoover assignment

6

Bernoulli Trial

- A process which
  - has two mutually exclusive outcomes: **success** and **failure**
  - P(success) is constant over a series of trials
- Examples
  - a coin toss
  - each Guess Your Best quiz question
  - each Hoover sales call

7

Binomial Distribution

- Used to determine the probability of a given # of successes in a given # of BTRIALS
- Although the result of any individual trial is uncertain, the Binomial distribution accurately predicts the distribution of the number of successes over a series of BTRIALS
- Discrete… why?

8

Binomial Formula

- When involved with a series of BTRIALS, the corresponding probability tree has a special structure which allows streamlined calculations

$$p_B(x|n,p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)}$$

- Hoover assignment
- Convenience Store | IRS | Basketball | Financial Aid

9

Binomial Tables Booklet

- n=1, 2, 3, 4,.., 20 p=.01, .05, .10, .15,..., .95
- Hoover scenario: n = 3, p = .20
  - $P_B(X=0 \mid n=3, p=.20)=$
  - $P_B(X=1 \mid n=3, p=.20)=$
  - $P_B(X=2 \mid n=3, p=.20)=$
  - $P_B(X=3 \mid n=3, p=.20)=$

  - Convenience Store | IRS | Basketball
  - Ch5 Wks 1, problem 3a-c

Mean And Variance Of Binomial Distribution

- Because of Binomial distribution's special structure:

$$E_B(x) = \mu_B = np$$

$$\sigma^2_B = np(1-p)$$

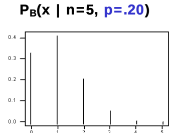
- Hoover assignment
- Convenience Store | IRS | Basketball

Binomial Distribution Shape (Ch7)

- The value of **p** determines the distribution's skew

- when p<.50 BINO is positively skewed

$P_B(x \mid n=5, p=.20)$

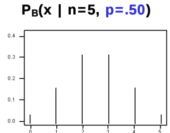


E(X) = 5(.2) =1

σ^2 = 5(.2)(.8) = 0.8

- when p=.50 BINO is symmetrical

$P_B(x \mid n=5, p=.50)$

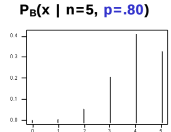


E(X) = 5(.5) =2.5

σ^2 = 5(.5)(.5) = 1.25

- when p>.50 BINO is negatively skewed

$P_B(x \mid n=5, p=.80)$



E(X) = 5(.8) = 4

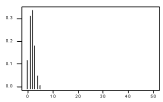
σ^2 = 5(.8)(.2) = 0.8

- Ch5 Wks 1, problem 3a-c

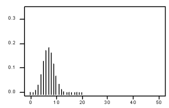
Binomial Distribution Shape (Ch7)

- As the value of **n** increases, the BINO distribution becomes more symmetrical, regardless of p
  - as n ↑, the Binomial distribution approaches the Normal distribution

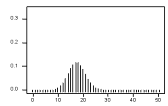
$P_B(x \mid n=5, p=.35)$



$P_B(x \mid n=20, p=.35)$



$P_B(x \mid n=50, p=.35)$



- When $np \geq 5$ and $n(1-p) \geq 5$, the Binomial distribution can be approximated by a Normal distribution

13

Poisson Distribution

$$P_p(x|\mu) = \frac{\mu^x e^{-\mu}}{x!}$$

- Used to determine the probability of a given number of successes which occur over a continuum of time or space
- A discrete distribution… why?
- Two major assumptions:
  - probability of occurrence for an event is constant for any two intervals of equal length
  - the occurrence/nonoccurrence in any interval is independent of the occurrence/nonoccurrence in any other interval
- Must know (or estimate) m , which represents _____
- Caution when question's units of measurement don't match the mean's units of measurement … rescale the mean to match the question

14

Poisson Distribution

- Poisson probability Tables
- Maytag Problem

- Because of Poisson's special structure (nib)

$$E_P(x) = \mu \qquad \sigma^2_P(x) = \mu$$

- Maytag:

  E(calls/hr) = 1.5 $\qquad$ σ^2(calls/hr) = 1.5

15