Chapter 4
Introduction to Probability

Decision Making and Uncertainty

- Managers can make better decisions when they're able to estimate an event's probability of occurring
- Probability
  - a value ranging from 0 to 1 that indicates an event's probability of occurring

Experiments and Random Variables

- Experiment
  - a process that generates well-defined outcomes
- The result of an experiment is a random variable
  - a process that:
    1) has more than 1 possible outcome and
    2) you don't know which outcome until after it occurs
- Life is full of random variables...

Sample Space

- The entire set of all possible outcomes of an experiment
- Example: a couple plans 2 children
  - sample space consists of 4 possible distinct outcomes $\{B_1B_2, B_1G_2, G_1B_2, G_1G_2\}$
- Elementary event (sample point)
  - an individual outcome of an experiment
- Event
  - a collection of elementary events of interest
    - event same gender includes $\{B_1B_2, G_1G_2\}$
    - event no boys includes $\{G_1G_2\}$
    - event at least one boy includes $\{B_1B_2, B_1G_2, G_1B_2\}$
    - event boy born first includes $\{B_1B_2, B_1G_2\}$
- Composite event (nib)
  - an event that consists of more than 1 elementary event
    - i.e., an event that can occur more than one way
  - of the events above, which are composite events?

3 General Approaches to Defining Probability

- Classical (a priori)
  - iff all possible outcomes are equally likely, then:

$$P(E) = \frac{\text{number of ways event can occur}}{\text{total possible equally likely outcomes}} = \frac{a}{(a+b)}$$

  - examples: P(Ace) = P(Heads) =
- Relative Frequency (Empirical) approach
  - observe many trials and record how frequently the event of interest occurs
  - use this history to estimate event's tendency to occur in the future

$$P(E) \approx \frac{\text{number of past occurrences}}{\text{total number of observations}}$$

  - example: you're given a 'doctored' coin, how estimate P(heads)?
  - 2 assumptions this approach is based on
    -
    -
- Subjective (personalistic)
  - degree of belief that the event of interest will occur
  - can't observe; no past
  - example: P(referendum will pass)=___

Counting Techniques

- Used to determine how many subsets can be made from an original set of objects
  - Multiple-step experiments
  - Permutations
  - Combinations
- Helpful in determining the denominator when calculating probabilities using the classical approach to defining probability

Multiple-Step Experiments

- Used to determine the number possible outcomes (sequences) where there are multiple steps and each step has a set of possible outcomes.

$$\#\text{ of outcomes} = (n_1)(n_2)\ldots(n_k)$$

- Tree Diagrams help portray the multi-step experiment and organize givens
- Power Train assignment

Permutation

- The number of possible ordered arrangements where selection is made without replacement.

$$P_n^N = \frac{N!}{(N-n)!}$$

- Executive Committee assignment

Combination

- The number possible outcomes where order of selection is not important and selections are made without replacement.

$$C_n^N = \frac{N!}{n!(N-n)!}$$

- Executive Committee assignment
- Lotto assignment

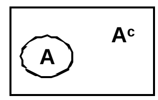
Assigning Probabilities

- P(E) "the probability of event E"

- Basic requirements for assigning probabilities:
  - $0 \leq P(E_i) \leq 1$
  - $\Sigma P(E_i) = 1$

Events

- Event
  - a collection of elementary events of interest
- Probability of an Event
  - denoted P(A)
  - the sum of the probabilities of the elementary events which lead to the event's occurrence
- Complement of an Event



  - denoted A^c **or** A
  - collection of elementary events which don't lead to event A's occurrence
  - example: at least one boy and no boys {B_1B_2, B_1G_2, G_1B_2, G_1G_2}
- Complement Rule
  - collectively exhaustive

$$P(A) = 1 - P(A^c)$$

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Contingency Table

- Review from Chapter 2
  - used to simultaneously cross-classify **two variables**
  - can reveal interactions between variables
  - can contain frequencies or relative frequencies
- Helps organize the givens in probability problems
- Job Applicant, part a

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3 Basic Types of Probabilities

- Marginal (Simple) Probability
  - probability of an event occurring **without regard** to any other event's occurrence
  - denoted: P(A)
  - Job Applicant, part b
- Joint Probability
  - probability of 2 (or more) events occurring **together** (or in sequence)
  - denoted: $P(A \cap B)$
  - Job Applicant, part c
- Conditional Probability
  - probability of an event occurring **given that** another event has already occurred
  - relevant only when involved with **dependent events**
  - Job Applicant, part d

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

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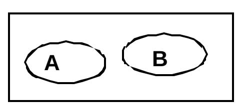
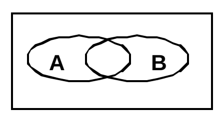
Events





- Mutually Exclusive Events
  - cannot occur together (jointly)
  - occurrence of one event **precludes** the occurrence of the other
  - events which have no elementary outcomes in common
  - examples: club & heart; male & female; boy first & no boys

$\{B_1B_2,\ B_1G_2,\ G_1B_2,\ G_1G_2\}$

- Nonexclusive Events (nib)
  - the two events can occur together, but they don't have to
  - events which have at least one elementary outcome in common
  - egs: ace & heart; male & awake; same sex & at least one boy

$\{B_1B_2,\ B_1G_2,\ G_1B_2,\ G_1G_2\}$

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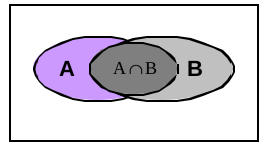
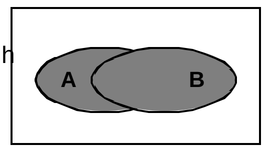
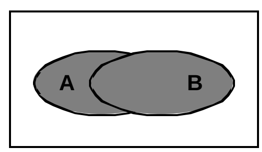
Events

- Intersection of Events (AND)
  - the **joint** occurrence of 2 (or more) events occurring either together or in sequence
  - denoted: A Ç B
  - example: P(Ace and Heart)?



- Union of Events (OR)
  - the occurrence of one event **or** the other **or** both
  - denoted: A È B
  - example 1: P(Ace or Heart)?





Rule(s) of Addition

- Used to determine the probability of one event **or** the other **or** both occurring
  - i.e., the probability of the **union** of 2 events, $P(A \cup B)$
- Although the author shows 2 rules of addition, we only need 1

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - \boxed{P(A \cap B)}$$

- Special emphasis on the last term
  - why do we need it?
  - when the events are mutually exclusive, $P(A \cap B) = 0$
- Example 1: P(Ace U Heart)=
- Example 2: P(Ace U King)=
- Job Applicant assignment, part f

Events

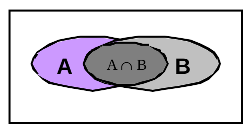
- Dependent Events
  - occurrence of one event **changes** (↑ or ↓) the prob of occurrence of the other
  - example: You're dealt 2 cards face down, without replacement.
    - what's probability the 1st is an ace?
    - what's probability the 2nd is an ace?
  - need **conditional probabilities** to express the probability of an event's occurrence under the different possible conditions

$$P(A_2 \mid A_1) = \_\_\_\_ \qquad P(A_2 \mid A^c_1) = \_\_\_\_$$

- Independent Events
  - iff occurrence of one event has **no effect** on prob of occurrence of the other
  - example: You're dealt 2 cards face down, **with replacement**.
    - what's probability the 1st is an ace?
    - what's probability the 2nd is an ace?

$$P(B \mid A) = P(B \mid A^c) = P(B)$$

Conditional probability formula, repackaged



Rules of Multiplication

- Used to determine the probability of the **joint** occurrence of 2 (or more) events.

$$P(A \text{ and } B) = P(A \cap B) = P(A) * P(B) \quad \text{if events are independent}$$

$$P(A \text{ and } B) = P(A \cap B) = P(A) * P(B \mid A) \quad \text{if events are dependent}$$

- When events A and B are independent, then P(B|A) = P(B) and these rules are equivalent
- Assignment: Deal 2 cards **with** replacement.
- Assignment: Deal 2 cards **without** replacement.
- Burglaries assignment
- Collins assignment

Testing Statistical Independence (nib)

- Method 1
  - Use the rule of multiplication for independent events. If it holds, the events are statistically independent.

$$P(A \cap B) \stackrel{?}{=} P(A) * P(B)$$

- Method 2
  - Contrast an appropriate set of **conditional** probabilities. If they're equal, the two events are statistically independent.

$$P(B \mid A) \stackrel{?}{=} P(B \mid A^c)$$

- Job Applicant assignment, part e
- Collins Company assignment, part b

Bayes' Theorem

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

- Conditional probability with a chronological twist
- Used to determine the probability for the first of two sequential events given that the second event's outcome has already been observed
- Prior Probability
  - probability of an event's occurrence **before** additional information acquired
- Posterior Probability
  - a revised probability **after** acquiring additional information

- Collins assignment, part j
- Pro Football assignment