Lesson 10 – Radical Functions

In this lesson, we will learn some new properties of exponents, including those dealing with Rational and Radical Roots. We will also revisit complex numbers.

Our function type for this lesson is Radical Functions and you will learn their characteristics, their graphs, and you will solve their equations both graphically and algebraically.

Lesson Topics

Lesson 10 Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Name: ____________________ **Date:** ________________

Mini-Lesson 10

Section 10.1 – Roots, Radicals, and Rational Exponents

SQUARE ROOTS

The square root of a is written as $\sqrt{a}$. If $\sqrt{a} = b$ then $b^2 = a$.

NOTATION:

The notation $\sqrt{a}$, is RADICAL NOTATION for the square root of a.

The notation $a^{\frac{1}{2}}$ is RATIONAL EXPONENT NOTATION for the square root of a.

On your TI 83/84 calculator, you can use the $\sqrt{}$ symbol to compute square roots.

EXAMPLES:

a) $\sqrt{25} = 5$ because $5^2 = 25$

b) Note that the square and square root "undo" each other)

$$\left(\sqrt{144}\right)^2 = 144 \text{ and } \sqrt{144^2} = 144$$

c) $25^{1/2} = \sqrt{25} = 5$

d) $\sqrt{-64} = (-64)^{1/2}$ is not a real number because there is no number, squared, that will give -64

THE NTH ROOT

$\sqrt[n]{a} = a^{1/n}$, the nth root of a.

EXAMPLES:

a) $\sqrt[4]{256} = 256^{1/4} = 4$ Calculator Entry: 256^(1/4)

b) $\sqrt[7]{-2187} = (-2187)^{1/7} = -3$ Calculator Entry: (-2187)^(1/7)

c) $-\sqrt[3]{15} = -15^{1/3} \approx -2.47$ Calculator Entry: -15^(1/3)

d) $\sqrt[6]{-324} = (-324)^{1/6}$ is not a real number Calculator Entry: (-324)^(1/6)

RATIONAL EXPONENTS

$$a^{p/q} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$$

EXAMPLES:

a) $\left(\sqrt[4]{81}\right)^3 = (81)^{3/4} = (3)^3 = 27$

b) $\left(\sqrt[3]{-125}\right)^2 = (-125)^{2/3} = (-5)^2 = 25$

c) $\dfrac{1}{\left(\sqrt[5]{32}\right)^2} = \dfrac{1}{32^{2/5}} = 32^{-2/5} = \dfrac{1}{2^2} = \dfrac{1}{4}$

Problem 1	**MEDIA EXAMPLE – Compute with Rational/Radical Exponents**

Compute each of the following showing as much work as possible. Round to two decimal places as needed. Check results using your calculator.

a) $\sqrt{49} =$

b) $\sqrt[3]{8} =$

c) $\sqrt{-49} =$

d) $\sqrt[3]{-8} =$

e) $-25^{3/2}$

f) $(-25)^{3/2}$

g) $\sqrt[7]{49}$

h) $\sqrt[4]{12^3}$

Problem 2	**YOU TRY – Compute with Rational/Radical Exponents**

Compute each of the following showing as much work as possible. Round to two decimal places as needed.

a) $\sqrt{36}$

b) $\sqrt[3]{-64}$

c) $16^{3/2}$

d) $(-25)^{1/2}$

e) $\left(\sqrt[3]{27}\right)^{4}$

f) $\sqrt[9]{81}$

Section 10.2 – Square Root Functions – Key Characteristics

A basic square root function has the form

$$f(x)=\sqrt{p(x)},$$

where $p(x)$ is a polynomial, and $p(x)\geq 0$.
(Remember that we cannot take the square root of negative numbers in the real number system.)

DOMAIN
To determine the domain of $f(x)$, you want to find the values of x such that $p(x)\geq 0$.

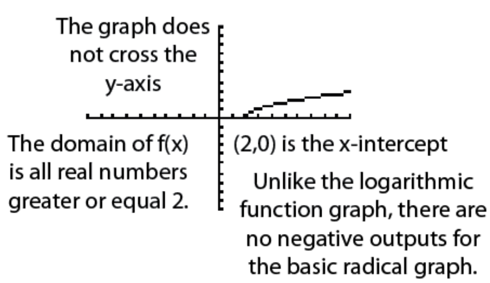
HORIZONTAL INTERCEPT
To determine the horizontal intercept for the basic square root function $f(x)=\sqrt{p(x)}$, solve the equation $p(x) = 0$.

VERTICAL INTERCEPT
To determine the vertical intercept, evaluate $f(0)$.

Problem 3	**WORKED EXAMPLE – Key Characteristics of Square Root Functions**

Graph $f(x)=\sqrt{x-2}$ and determine vertical intercept, horizontal intercept, and domain of f(x).

To graph, input into Y1 the following: 2nd>X^2 then x-2) so that Y1= $\sqrt{(x-2)}$. Graph on the standard window (Zoom 6) to get the graph below:



DOMAIN
Solve $x-2\geq 0$ to get $x\geq 2$.
Therefore the domain is $x\geq 2$.

HORIZONTAL INTERCEPT
Solve $x - 2 = 0$ to get $x = 2$. The horizontal intercept is (2,0)

VERTICAL INTERCEPT
Determine f(0) = $\sqrt{0-2}=\sqrt{-2}$ which is not a real number.
So, there is no vertical intercept.

Problem 4	**MEDIA EXAMPLE – Key Characteristics of Square Root Functions**

For each of the following, determine the domain, horizontal intercept, and vertical intercept, then sketch an accurate graph of each.

a) $f(x) = \sqrt{4 - x}$

Domain of $f(x)$:

Horizontal Intercept:

Vertical Intercept:

b) $g(x) = \sqrt{x - 4}$

Domain of $g(x)$:

Horizontal Intercept:

Vertical Intercept:

Problem 5	**YOU TRY – Key Characteristics of Square Root Functions**

Given the function $f(x) = \sqrt{12 - 4x}$, determine the domain, the horizontal intercept, the vertical intercept (if it exists), and draw an accurate graph of the function. Round to one decimal place as needed.

Domain of $f(x)$:

Horizontal Intercept:

Vertical Intercept:

Section 10.3 – Cube Root Functions – Key Characteristics

A basic cube root function has the form

$$f(x) = \sqrt[3]{p(x)},$$

where $p(x)$ is a polynomial.

DOMAIN
The domain of $f(x)$ is all real numbers.

HORIZONTAL INTERCEPT
To determine the horizontal intercept for the basic cube root function $f(x) = \sqrt[3]{p(x)}$, solve the equation $p(x) = 0$.

VERTICAL INTERCEPT
To determine the vertical intercept, evaluate $f(0)$.

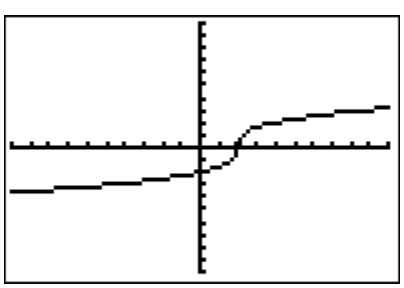
Problem 6	**WORKED EXAMPLE – Key Characteristics of Cube Root Functions**

Graph $f(x) = \sqrt[3]{4x-8}$ and determine vertical intercept, horizontal intercept, and domain of $f(x)$.

To enter $f(x)$ into your calculator, first rewrite the radical as a rational exponent:

$$\sqrt[3]{4x-8} = (4x-8)^{1/3}$$

Graph on the standard window (Zoom 6) to get the graph below:



DOMAIN
The domain is all real numbers.

HORIZONTAL INTERCEPT
Solve $4x - 8 = 0$ to get $x = 2$. The horizontal intercept is (2,0)

VERTICAL INTERCEPT
$f(0) = \sqrt[3]{4(0)-8} = \sqrt[3]{-8} = -2$
So the vertical intercept is (0, –2).

Problem 7	**MEDIA EXAMPLE – Key Characteristics of Cube Root Functions**

For each of the following, determine the domain, horizontal intercept, and vertical intercept, then sketch an accurate graph of each.

a) $f(x)=\sqrt[3]{27-15x}$

Domain of $f(x)$:

Horizontal Intercept:

Vertical Intercept:

b) $g(x)=\sqrt[3]{2x-10}$

Domain of $g(x)$:

Horizontal Intercept:

Vertical Intercept:

Problem 8	**YOU TRY – Key Characteristics of Cube Root Functions**

Given the function $f(x)=\sqrt[3]{x+8}$, determine the domain, the horizontal intercept, the vertical intercept (if it exists), and draw an accurate graph of the function.

Domain of $f(x)$:

Horizontal Intercept:

Vertical Intercept:

Section 10.4 – Radical Functions – Key Characteristics

A basic radical function has the form $f(x)=\sqrt[n]{p(x)}$, where $n>0$ and $p(x)$ is a polynomial.

<u>DOMAIN</u>: The DOMAIN of $f(x)=\sqrt[n]{p(x)}$ depends on the value of n.

- If n is EVEN (like the square root function), then the domain consists of all values of x for which $p(x)\geq 0$. Remember that we cannot take an even root of negative numbers in the real number system.
- If n is ODD (like the cube root function), then the domain is all real numbers.

<u>HORIZONTAL INTERCEPT</u>

To determine the horizontal intercept for the basic cube root function $f(x)=\sqrt[n]{p(x)}$, solve the equation $p(x) = 0$.

<u>VERTICAL INTERCEPT</u>

To determine the vertical intercept, evaluate $f(0)$.

Problem 9	**MEDIA EXAMPLE – Key Characteristics of Radical Functions**

For each of the following, determine the domain, horizontal intercept, and vertical intercept, then sketch an accurate graph of each. Write your answers in exact form and give the decimal approximation rounded to the nearest hundredth.

a) $f(x)=\sqrt[4]{x-5}$

Domain:

Horizontal Intercept:

Vertical Intercept:

b) $g(x)=\sqrt[7]{11+x}$

Domain:

Horizontal Intercept:

Vertical Intercept:

Problem 10	**YOU TRY– Key Characteristics of Radical Functions**

For each of the following, determine the domain, horizontal intercept, and vertical intercept, then sketch an accurate graph of each. Write your answers in exact form and give the decimal approximation rounded to the nearest hundredth.

a) $f(x)=\sqrt[5]{20-x}$

Domain:

Horizontal Intercept:

Vertical Intercept:

b) $g(x)=\sqrt[8]{8+4x}$

Domain:

Horizontal Intercept:

Vertical Intercept:

Section 10.5 – Solve Radical Equations by Graphing

Solve Radical Equations by Graphing

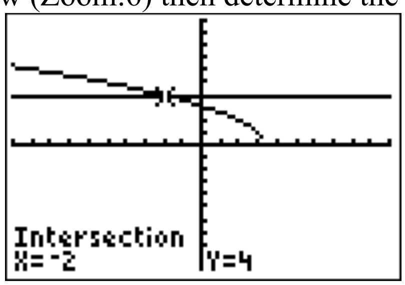
- Let Y1 = one side of the equation
- Let Y2 = other side of the equation
- Determine an appropriate window to see important parts of the graph
- Use the Intersection Method
- *Note: If your graphs do not cross, then there is no intersection and no solution to the equation.*

Problem 11	**WORKED EXAMPLE – Solve Radical Equations by Graphing**

Solve the equation $\sqrt{10-3x}=4$ graphically.

Let Y1 = $\sqrt{10-3x}$
Let Y2 = 4

Graph on the standard window (Zoom:6) then determine the intersection (seen below).



Your solution is the x-value of the intersection which in this case is $x = -2$.

Problem 12	**MEDIA EXAMPLE – Solve Radical Equations by Graphing**

Solve the equations graphically. Include a rough but accurate sketch of the graphs and intersection point. Mark and label the intersection. Round answers to two decimal places.

a) $\sqrt[3]{2x-1}=5$

Solution: ______________________

b) $41+5\sqrt{2x-4}=11$

Solution: ______________________

Problem 13	**YOU TRY – Solve Radical Equations by Graphing**

Solve the equation $\sqrt[5]{8x+133}=4$ graphically. Include a rough but accurate sketch of the graphs and intersection point. Mark and label the intersection. Round answers to two decimal places.

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

Solution: x = ______________

Section 10.6 – Solve Radical Equations Algebraically

To solve radical equations algebraically (also called symbolically):

- Isolate the radical part of the equation on one side and anything else on the other
- Sometimes you will have radicals on both sides. That is ok.
- Raise both sides of the equation to a power that will "undo" the radical (2nd power to get rid of square root, 3rd power to get rid of cube root, etc…)
- Solve.
- *Check your answer! Not all solutions obtained will check properly in your equation.*

Problem 14	**WORKED EXAMPLE – Solve Radical Equations Algebraically**

Solve the equation $\sqrt{10-3x}=4$ algebraically.

First, square both sides to remove the square root.

$$\sqrt{10-3x}=4$$
$$\left(\sqrt{10-3x}\right)^2=4^2$$
$$10-3x=16$$

Next, isolate *x*.

$$10-3x=16$$
$$-3x=6$$
$$x=-2$$

VERY IMPORTANT! Check $x = -2$ in the original equation to be sure it works! *Not all solutions obtained using the process above will check properly in your equation.* If an x does not check, then it is not a solution**.**

$$\sqrt{10-3(-2)}=4$$
$$\sqrt{10+6}=4$$
$$\sqrt{16}=4$$
$$4=4$$

$x = -2$ is the solution to this equation.

Problem 15	**MEDIA EXAMPLE – Solve Radical Equations Algebraically**

Solve the equations algebraically. Write your answers in exact form, then give the decimal approximation rounded to the nearest hundredth.

a) $\sqrt[3]{2x-1}=5$

b) $41+5\sqrt{2x-4}=11$

Problem 16	**YOU TRY – Solve Radical Equations Algebraically**

Solve the equations algebraically. Write your answers in exact form, then give the decimal approximation rounded to the nearest hundredth. Be sure to check your final result!

a) $3\sqrt{4-x}-7=20$

b) $5+\sqrt[4]{3x-1}=7$

c) $2\sqrt{5x}+24=4$

d) $\sqrt[3]{2-5x}-4=6$

Problem 17	**WORKED EXAMPLE – Solve Radical Equations – More Advanced**

Solve the equation algebraically and check graphically: $\sqrt{x+6}=x$. Be sure to check your final result!

Since the radical is isolated, square both sides to remove the square root. Then, isolate x.

$$\sqrt{x+6}=x$$
$$\left(\sqrt{x+6}\right)^2=x^2$$
$$x+6=x^2$$
$$0=x^2-x-6$$
$$x^2-x-6=0$$

What we now have is a quadratic equation. The easiest and fastest way to work with this problem is through factoring. You can also use the Quadratic Formula or graphing.

$$x^2-x-6=0$$
$$(x+2)(x-3)=0$$
$$x+2=0 \text{ or } x-3=0$$
$$x=-2 \text{ or } x=3$$

CHECK:

When $x=-2$

$$\sqrt{-2+6}=-2?$$
$$\sqrt{4}=-2?$$
$$2\neq-2$$

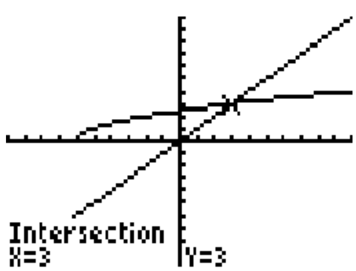
$x=-2$ **does not check so is not a solution**

When $x=3$

$$\sqrt{3+6}=3?$$
$$\sqrt{9}=3?$$
$$3=3$$

$x=3$ checks so is a solution.

Graphical Check: $Y1=\sqrt{x+6}$, $y2=x$ Window: Standard (Zoom:6)



Using the Intersection Method, we obtain a verified solution of $x=3$.

Problem 18	**MEDIA EXAMPLE – Solve Radical Equations – More Advanced**

Solve the equation algebraically ***and*** graphically: $1+\sqrt{7-x}=x$.
Be sure to check your final result!

Problem 19	**YOU TRY – Solve Radical Equations – More Advanced**

Solve the equation algebraically ***and*** graphically: $\sqrt{x+6}=x+4$.
Be sure to check your final result!