# Unit 10:  The Equation of a Linear Function

**Section 10.1:**  The Equation of a Linear Function

**Section 10.2:**   Writing Linear Equations in Slope-Intercept Form

**Section 10.3:**   Parallel and Perpendicular Lines

**Section 10.4:**   Applications – Slope-Intercept Form

**Section 10.5:**   Interpreting a Linear Function in Slope-Intercept Form

| KEY TERMS AND CONCEPTS | |
|---|---|
| Look for the following terms and concepts as you work through the Media Lesson.  In the space below, explain the meaning of each of these concepts and terms *in your own words*.  Provide examples that are not identical to those in the Media Lesson. | |
| Slope-Intercept Form | |
| How to Graph a Linear Equation given in Slope-Intercept Form | |
| How to Write the Equation of  a Line in Slope-Intercept Form given two points. | |
| Slopes of Parallel Lines | |

| | |
|---|---|
| Slopes of Perpendicular Lines | |
| The slope of a Vertical Line | |
| The Equation of a Vertical Line | |

# Unit 10:  Media Lesson

# Section 10.1:  The Equation of a Linear Function

| Slope – Intercept Form |
|:---:|

SLOPE-INTERCEPT FORM:
$$y = mx + b$$
$$y = b + mx$$
$$f(x) = mx + b$$

| Slope | Behavior |
|:---:|:---:|
| $m > 0$ | Increasing |
| $m < 0$ | Decreasing |
| $m = 0$ | Horizontal |
| $m$ is undefined | Vertical |

**Example 1**:   Fill in the table below.

| Equation | Slope | I, D, H, V | Vertical Intercept |
|:---:|:---:|:---:|:---:|
| $y = 3x + 5$ | | | |
| $y = 8 - x$ | | | |
| $y = 2x$ | | | |
| $y = -8$ | | | |

**Example 2**:   Determine the *horizontal* intercepts of each of the following.

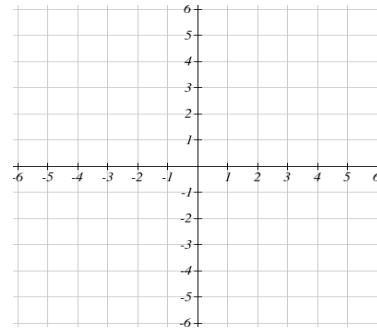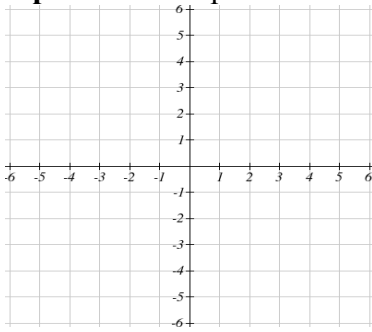$y = 3x + 5$ $\qquad\qquad$ $y = 8 - x$ $\qquad\qquad$ $y = 2x$ $\qquad\qquad$ $y = -8$

*To find a horizontal intercept*: _____

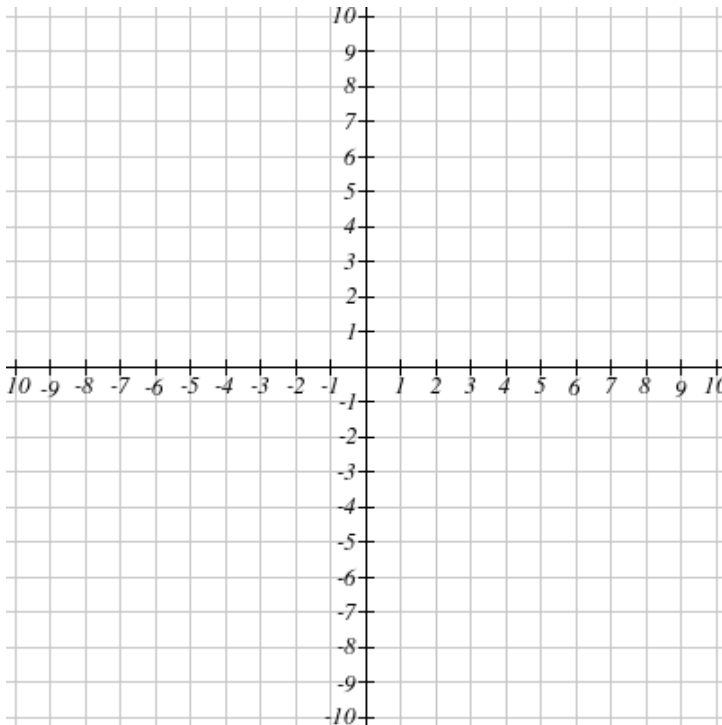**Example 3**:   The equation of a vertical line

**Example 4**:   Draw an **accurate** graph of the function  $f(x) = 4 - 3x$.

Slope:  _____

Vertical Intercept:  _____

Horizontal Intercept:  _____

*To find the Horizontal Intercept:*

_____

_____

Two additional points on the line:

_____   _____

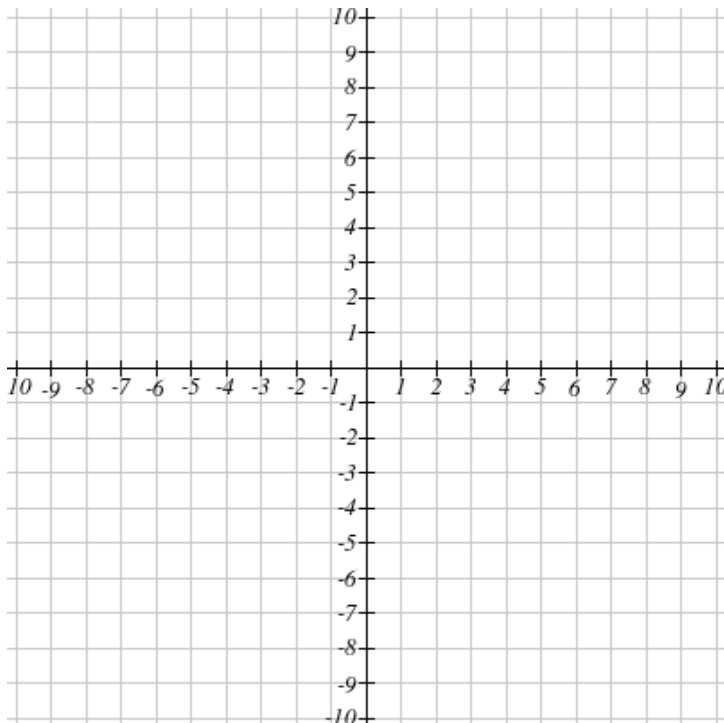| Slope-Intercept Form |
| :---: |
| $f(x) = mx + b$ |
| $f(x) = b + mx$ |

| Section 10.1 – You Try |
| :---: |

✏️ Complete the problems below.

a.  Fill in the table below.  Write intercepts as ordered pairs.  Write "DNE" if the answer does not exist.

| Equation | Slope | I, D, H, V | Vertical Intercept |
| :---: | :---: | :---: | :---: |
| $y = x - 11$ | | | |
| $G(x) = -2x$ | | | |
| $x = 5$ | | | |

I = Increasing, D = Decreasing,  H = Horizontal (Constant),  V = Vertical

b.  Draw an **accurate** graph of the function  $y = \dfrac{3}{4}x - 5$.  Identify the slope, intercepts, and two additional points on the line.



Slope:  _____

Vertical Intercept:  _____

Horizontal Intercept:  _____

Two additional points on the line:

_____    _____

# Section 10.2:  Writing the Equation of a Line
# in Slope-Intercept Form

| Slope-Intercept Form $y = mx + b$ |
|---|

**Example 1**:  Give the equation of the line in slope-intercept form

   a.  With vertical intercept  (0, 2) and  slope –9

   b.  Passing through (2, 3) with slope –5

   c.  Passing through (2, 6) and (4, 16)
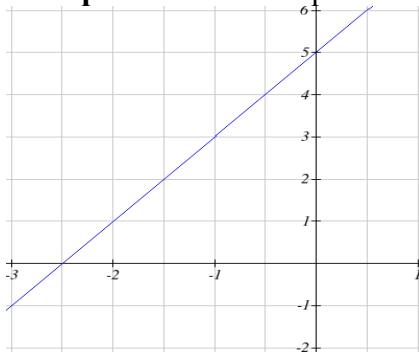
**Example 2**:  Give the equation of the linear function that would generate the following table of values.  Use your calculator to check.
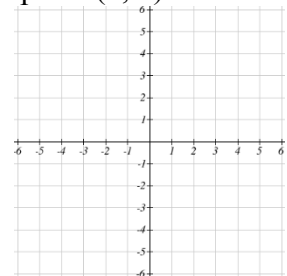
| $x$ | $f(x)$ |
|-----|--------|
| –5  | 238    |
| –3  | 174    |
| –1  | 110    |
| 1   | 46     |
| 7   | –146   |
| 12  | –306   |

**Example 3**:  Give the equation of the linear function shown below.

**Example 4**:  Give the equation of the horizontal line passing through the point (1, 3).

**Example 5**:  Give the equation of the vertical line passing through the point (1, 3).

| Section 10.2 – You Try |
|:---:|

Complete the problems below.  Show as much work as possible, as demonstrated in the Media Examples.

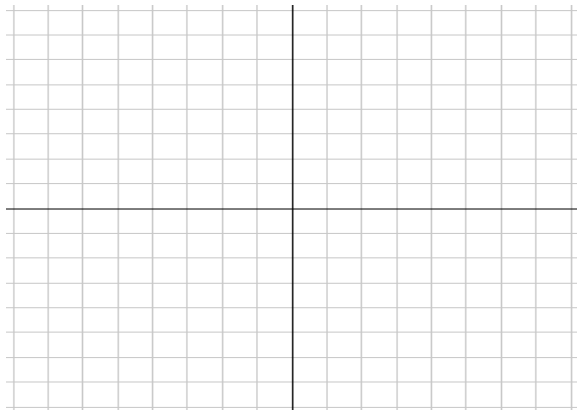a.   Give the equation of the line passing through the points (1, 7) and (3, –9).

b.   Give the equation of the horizontal line passing through the point (5, 11).
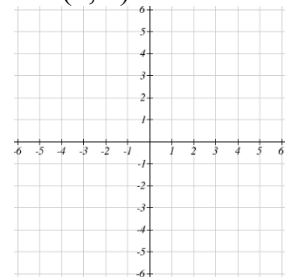
# Section 10.3:  Parallel and Perpendicular Lines

| Parallel Lines |
|---|

The slopes of Parallel Lines are _____



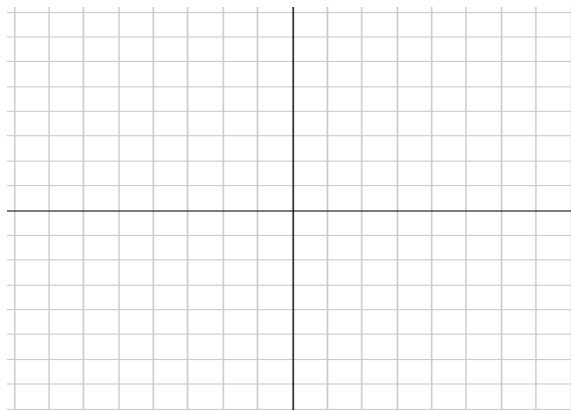| **Slope-Intercept Form** |
|---|

$$y = mx + b \qquad f(x) = mx + b$$

$$m = \text{slope}$$

$$b = \text{vertical intercept } (0,b)$$

**Example 1**:  Give the equation of the line passing through the point (8, 3) that is **parallel** to the line $y = -2x + 3$.

| Perpendicular Lines |
|---|

The slopes of perpendicular lines are _____

If Line 1 and Line 2 are perpendicular to each other, then

| Slope of Line 1 | Slope of Line 2 |
|:---:|:---:|
| $\dfrac{2}{3}$ | |
| 5 | |
| $-8$ | |
| $-\dfrac{4}{5}$ | |

<table>
<tr><td><b>Negative (Opposite)<br>Reciprocals</b><br><br>$\dfrac{a}{b}$ and $-\dfrac{b}{a}$</td></tr>
</table>

**Example 2**: Give the equation of the line passing through the point (8, 3) that is **perpendicular** to the line $y = -2x + 3$.

Give the equation of the line passing through the point (–3, 1) that is:

a. **Parallel** to the line $y = 8x - 5$.

b. **Perpendicular** to the line $y = 8x - 5$.

# Section 10.4:  Applications – Slope-Intercept Form

| Slope-Intercept Form | If we are not given the slope and vertical intercept, we need: |
|---|---|
| $y = mx + b$      $f(x) = mx + b$<br><br>$m$ = slope = rate of change<br><br>$b$ = vertical intercept (initial value) | • One point and the slope<br><br>• Two points |

**Example 1**:  You have just bought a new Sony 55" 3D television set for $2300.  The TV's value decreases at a rate of $250 per year.  Construct a linear function to represent this situation.

**Example 2**:  In 1998, the cost of tuition at a large Midwestern university was $144 per credit hour.  In 2008, tuition had risen to $238 per credit hour.  Determine a linear equation to represent the cost, C, of tuition as a function of *x,* the number of years since 1990.

| Section 10.4 – YOU TRY |
|---|

For each of the following, determine a linear equation to represent the given situation.  Use the indicated variables and proper function notation.  Show all possible steps.

a.  A tree is 3 feet tall when it is planted, and it grows by approximately half a foot each year.  Let $H(t)$ represent the height of the tree (in feet) after $t$ years.

b.  The enrollment at a local charter has been decreasing linearly.  In 2006, there were 857 students enrolled.  By 2015, there were only 785 students enrolled.  Let $S(n)$ represent the number of students enrolled in this school $n$ years after the year 2000.

# Section 10.5
# Interpreting a Linear Function in Slope-Intercept Form

**Example 1**:  The function $A(m) = 200 - 1.25m$  represents the balance in a bank account (in thousands of dollars) after $m$ months.

a.  Identify the slope of this linear function and interpret its meaning in a complete sentence.

b.  Identify the vertical intercept.  Write it as an ordered pair and interpret its practical meaning in a complete sentence.

Ordered Pair:_____

c.  Determine the horizontal intercept of this linear function.  Write it as an ordered pair and interpret its practical meaning in a complete sentence.

Ordered Pair:_____

d.  Determine $A(12)$.  Write your answer as an ordered pair and interpret its practical meaning in a complete sentence.

Ordered Pair:_____

e.  How long will it take for the balance in this account to reach $80,000?  Write the corresponding ordered pair.

Ordered Pair:_____

| Section 10.5 – You Try |
|---|

The function $E(t) = 3860 - 77.2t$  gives the surface elevation (in feet above sea level) of Lake Powell $t$ years after 1999.  Your answers must include all appropriate units.

a.  Identify the slope of this linear function and interpret its meaning in a complete sentence.

b.  Identify the vertical intercept.  Write it as an ordered pair and interpret its practical meaning in a complete sentence.

                                                                    Ordered Pair:_____

c.  Determine $E(5)$.  Write your answer as an ordered pair and interpret its practical meaning in a complete sentence.   Show your work.

                                                                    Ordered Pair:_____

# Unit 10: Practice Problems

1. Determine the slope, behavior (increasing, decreasing, constant, or vertical), and vertical intercept (as an ordered pair) of each of the following. Write "DNE" if an answer does not exist.

| Equation | Slope | Behavior | Vertical Intercept |
|---|---|---|---|
| $y = x - 2$ | | | |
| $f(a) = 6 - 4a$ | | | |
| $P(n) = 3n$ | | | |
| $y = 4$ | | | |
| $x = 7$ | | | |
| $y = \dfrac{3}{5}x - 4$ | | | |
| $y = x$ | | | |
| $B(x) = 8 - x$ | | | |
| $V(t) = -70$ | | | |

**2.** Determine the horizontal intercepts for each of the following.  Write "DNE" if there is no horizontal intercept.
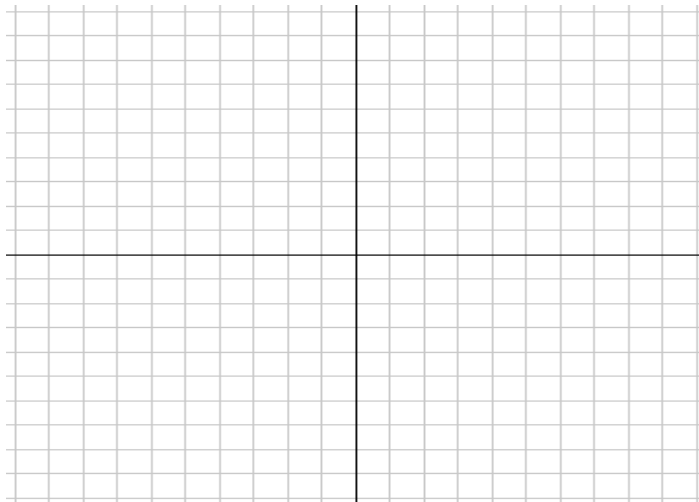
a. $y = x - 2$

b. $f(a) = 6 - 4a$

c. $P(n) = 3n$

d. $y = 4$

e. $x = 7$

f. $y = \dfrac{3}{5}x - 4$

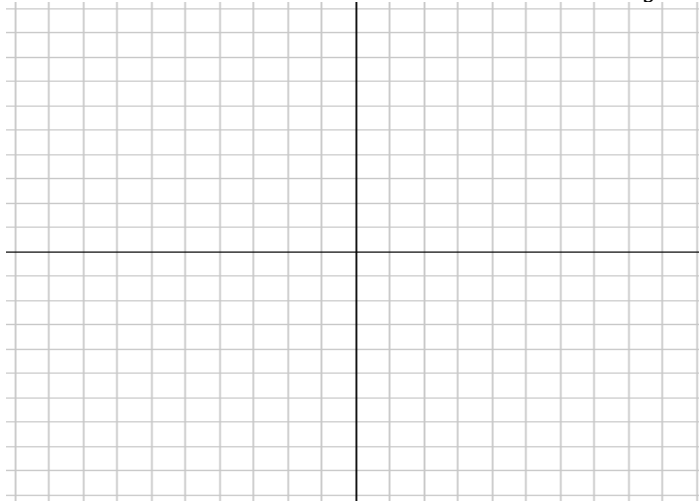3.  Draw an **accurate** graph of the function $f(x) = 4x + 5$.



Slope: _____

Vertical Intercept: _____

Horizontal Intercept: _____

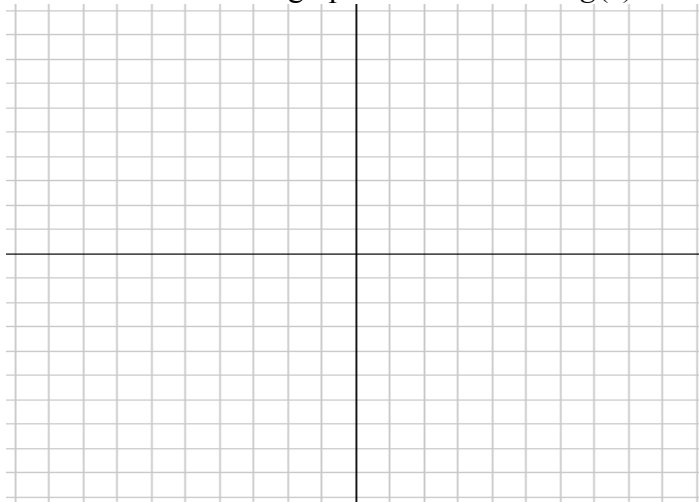**4.** Draw an **accurate** graph of the function $y = \frac{2}{5}x - 3$

Slope: _____

Vertical Intercept: _____

Horizontal Intercept: _____

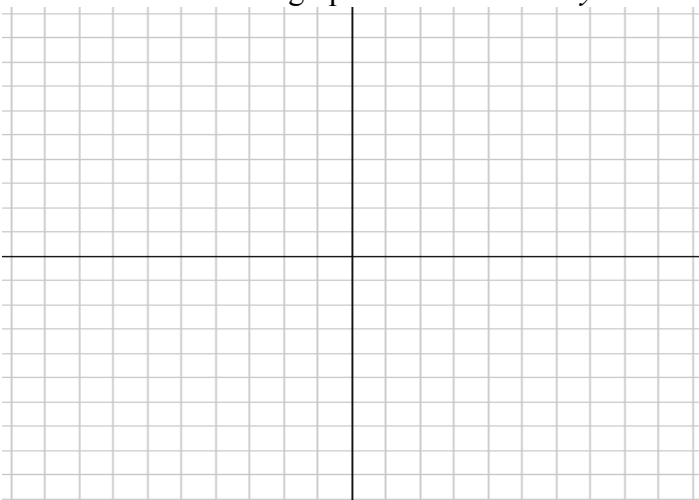**5.** Draw an **accurate** graph of the function $g(x) = 3 - x$.

Slope: _____

Vertical Intercept: _____

Horizontal Intercept: _____

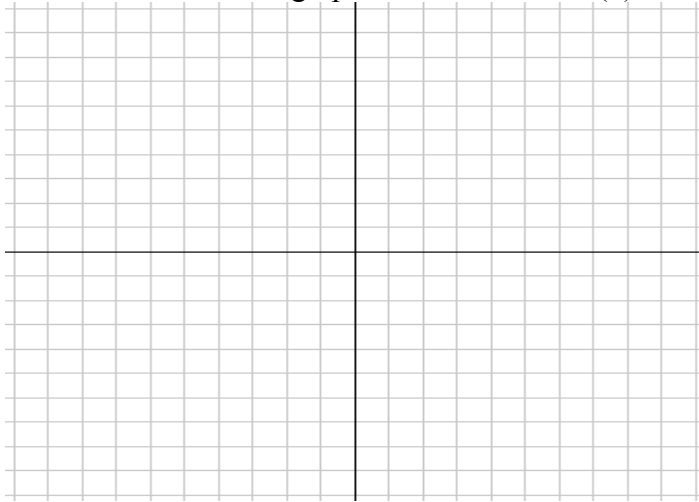**6.** Draw an **accurate** graph of the function $y = -2x$.

Slope: _____

Vertical Intercept: _____

Horizontal Intercept: _____

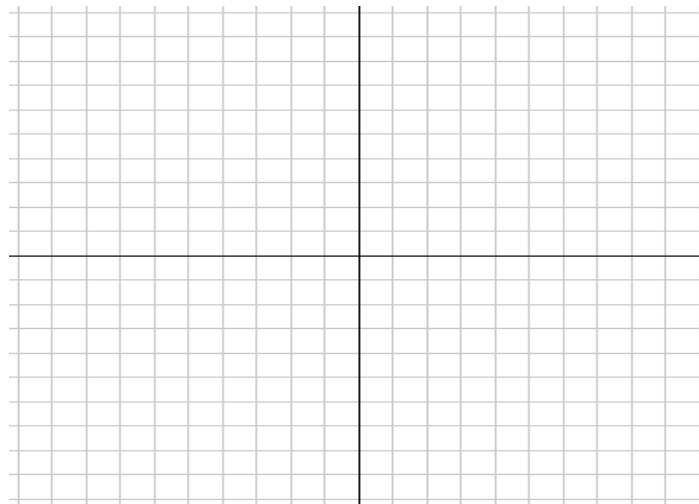**7.** Draw an **accurate** graph of the function $r(a) = 5$.

Slope:  _____

Vertical Intercept:  _____

Horizontal Intercept:  _____

**8.** Draw an **accurate** graph of the function $C(x) = \dfrac{x}{5}$

Slope:  _____

Vertical Intercept:  _____

Horizontal Intercept:  _____

**9.** Draw an **accurate** graph of the function $y = x$.
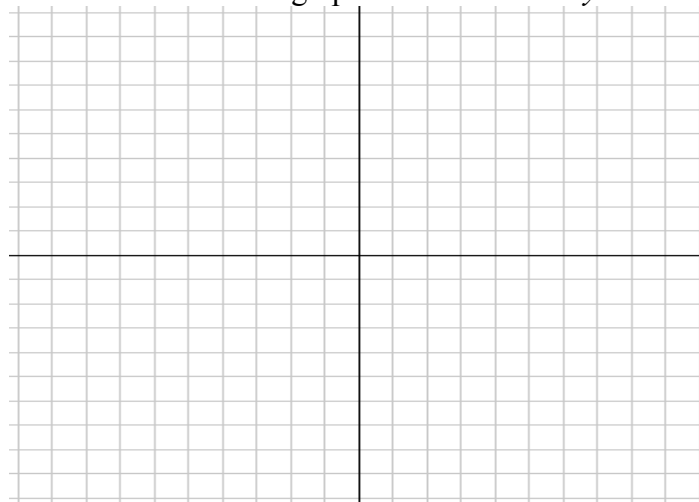
Slope:  _____

Vertical Intercept:  _____

Horizontal Intercept:  _____

**10.** Determine the equation of the line between each of the following pairs of points.

a.  (4, –5) and (2, 3)                    b.  (–3, 2) and (1, 8)

c.  (5, –9) and (5, 2)                    d.  (2, –1) and (–2, 3)

e.  (4, 3) and (12, –3)                    f.  (2, –4) and (7, –4)

11. Give the equation of the linear function that generates the following table of values.  Write
    your answer in slope-intercept form.

| $x$ | $f(x)$ |
|-----|--------|
| –5  | 91     |
| –2  | 67     |
| 1   | 43     |
| 4   | 19     |
| 9   | –21    |

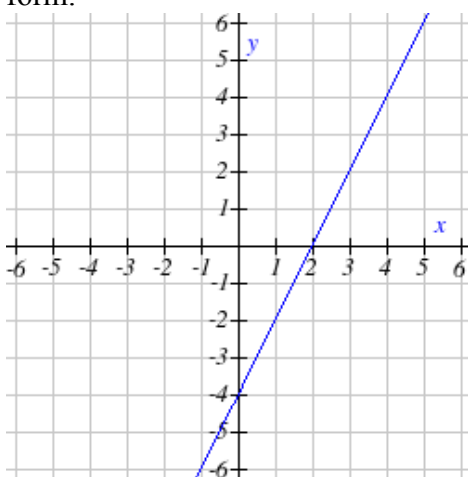12. Give the equation of the linear function that generates the following table of values.  Write
    your answer in slope-intercept form.

| $t$ | $C(t)$ |
|-----|--------|
| 5   | –1250  |
| 15  | –900   |
| 20  | –725   |
| 35  | –200   |
| 45  | 150    |

13. Give the equation of the linear function shown below.  Write your answer in slope-intercept
    form.

14. Give the equation of the linear function shown below.
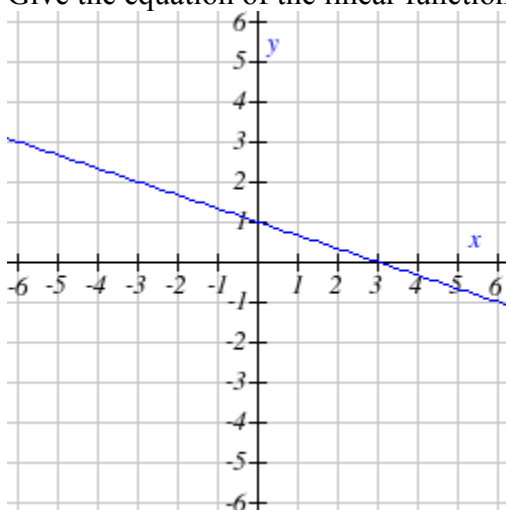


15. Give the equation of the linear function shown below.



16. Give the equation of the linear function shown below.
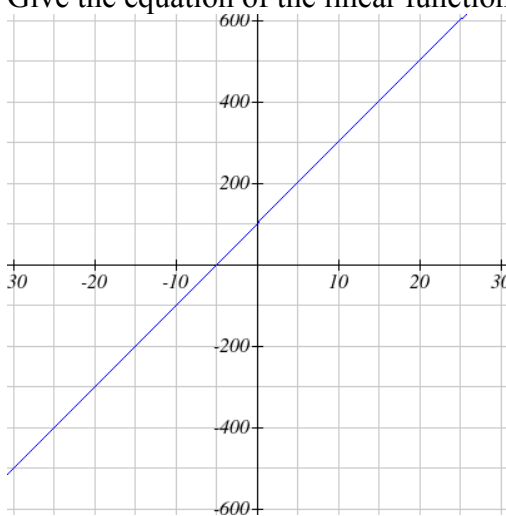
17. Give the equation of the linear function shown below.  Write your answer in slope-intercept form.
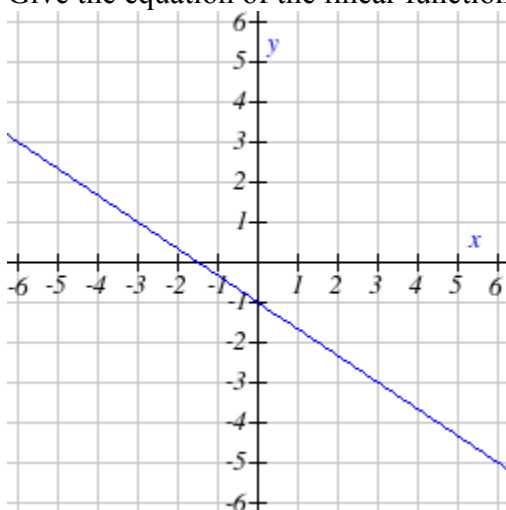


18. Give the equation of the linear function shown below.  Write your answer in slope-intercept form.



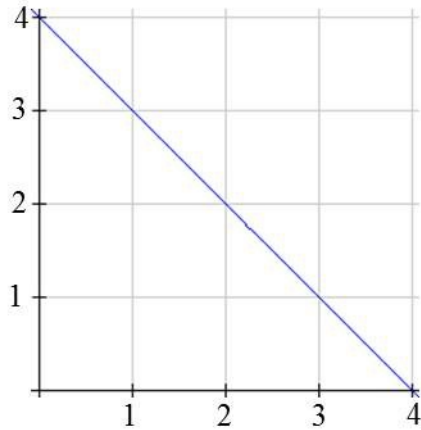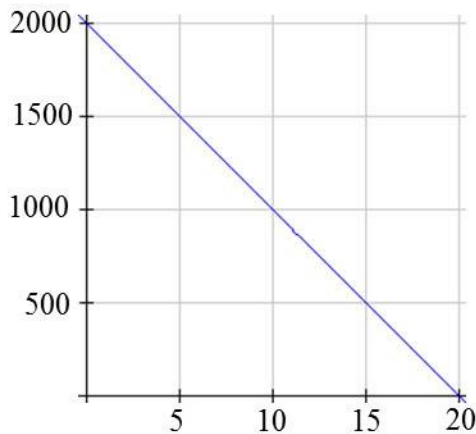19. Give the equation of the horizontal line passing through the point (–6, 11). _____

20. Give the equation of the vertical line passing through the point (4, 7). _____

21. Give the equation of the *x*-axis.  _____

22. Give the equation of the *y*-axis.  _____

23. Give the equation of the line passing through the point $(1, -5)$ that is parallel to $y = 12 - 8x$.

24. Give the equation of the line passing through the point $(4, 0)$ that is parallel to $y = 9 - \frac{3}{2}x$.

25. Give the equation of the line passing through the point $(10, 3)$ that is perpendicular to
$y = \frac{2}{5}x + 1$.

26. Give the equation of the line passing through the point $(-12, -1)$ that is perpendicular to
$y = 3 - 4x$.

| Applications |
|---|

27. A candy company has a machine that produces candy canes.  The number of candy canes produced depends on the amount of time the machine has been operating.  The machine produces 160 candy canes in five minutes.  In twenty minutes, the machine can produce 640 candy canes.

    a.  Determine the equation of the linear function that represents this situation.  Let $C(x)$ represent the number of candy canes produced in $x$ minutes.  Write your answer in function notation.

    b.  Determine $C(10)$.  Write a sentence explaining the meaning of your answer.

    c.  What is the practical meaning of the slope of this linear function?  Include units.

    d.  Determine horizontal intercept of this linear function.  Write it as an ordered pair and interpret its meaning.

    e.  How many candy canes will this machine produce in 1 hour?

28. Your workplace is 20 miles from your house.  The graph below shows the distance you are from your house if you leave work and drive in the opposite direction.
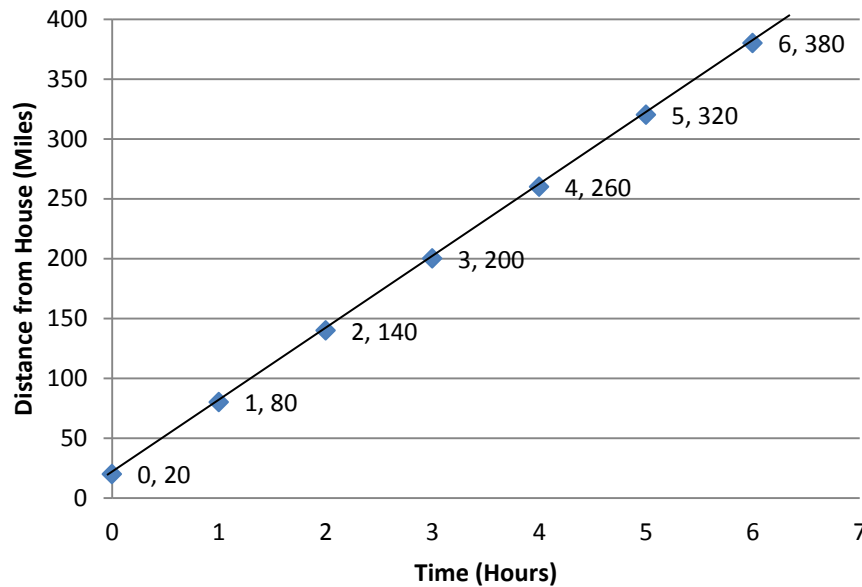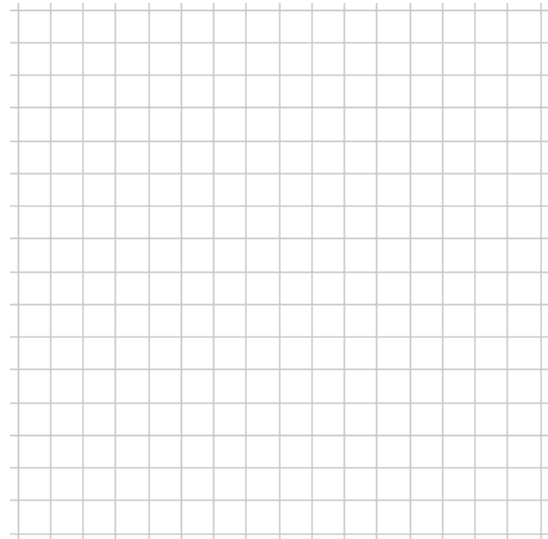


a.  Determine the equation of the linear function that represents this situation.  Let D(*t*) represent your distance from home after *t* hours.  Write your answer in function notation.

b.  Use the equation from part a to determine how long it would take for you to be 500 miles from your house.  Express your answer in hours and minutes.

c.  How far from your house would you be after 12 hours?

d.  Interpret the meaning of the slope of this linear function.

29. A local carpet cleaning company charges $10 for each room plus a reservation fee of $25. They clean a maximum of 12 rooms.   Also, they have the policy that once a reservation is made, if you cancel, the reservation fee is non-refundable.
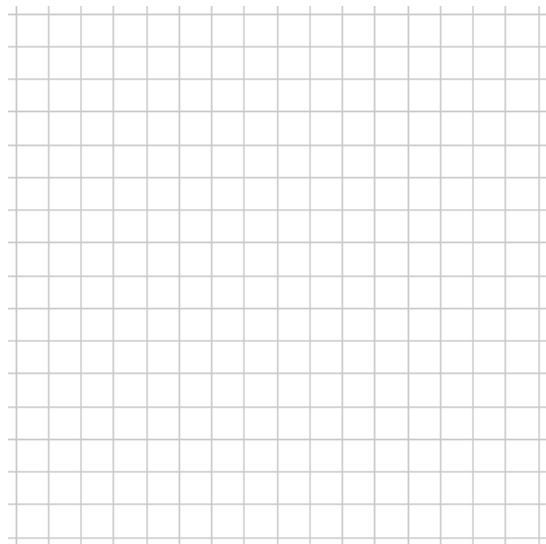
   a.  Determine the equation of the linear function $C(n)$ that represents the total cost for cleaning $n$ rooms.

   b.  Complete the table below.  Graph the results, and decide if it would make sense to connect the data points on the graph.

| $n$ | $C(n)$ |
|-----|--------|
| 0   |        |
| 1   |        |
| 2   |        |
| 3   |        |
| 6   |        |
| 12  |        |

30. Water is leaking out of a tank at a constant rate of 1 gallon every 2 minutes. The tank initially held 30 gallons of water.

a. Determine the equation of the linear function A(*t*) that represents the amount of water (in gallons) remaining in the tank after *t* minutes.

b. Complete the table below. Graph the results, and decide if it would make sense to connect the data points on the graph.

| *t* | A(*t*) |
|-----|--------|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 5 | |
| 10 | |
| 60 | |

c. Determine the practical domain of A(*t*):_____

d. Determine the practical range of A(*t*): _____

31. With good credit, and a $5000 down payment, you can finance a new 2012 Chevrolet
     Camaro convertible for 60 months for  $615.17 per month.
     a.  Determine the equation of the linear function, $T(n)$, that represents the total amount paid
         for this car after $n$ months.

     b.  Use the equation from part a to determine the total payment over the 60-month time
         period.

     c.  A new 2012 Chevrolet Camaro convertible has a base MSRP of $35,080.   Why is this
         value lower than your answer in part b?

32. The function $P(n) = 455n - 1820$ represents a computer manufacturer's profit when $n$ computers are sold.

    a.  Identify the slope, and interpret its meaning in a complete sentence.

    b.  Identify the vertical intercept.  Write it as an ordered pair and interpret its meaning in a complete sentence.

    c.  Determine the horizontal intercept.  Write it as an ordered pair and interpret its meaning in a complete sentence.

33. John is a door to door vacuum salesman.  His weekly salary is given by the linear function $S(v) = 200 + 50v$, where $v$ is the number of vacuums sold.

    a.  Identify the slope, and interpret its meaning in a complete sentence.

    b.  Identify the vertical intercept.  Write it as an ordered pair and interpret its meaning in a complete sentence.

34. The function $V(n) = 221.4 + 4.25n$ gives the value, in thousands of dollars, of an investment after $n$ years.

    a.  Identify the slope, and interpret its meaning in a complete sentence.

    b.  Identify the vertical intercept.  Write it as an ordered pair and interpret its meaning in a complete sentence.

35. The function $V(t) = 86.4 - 1.2t$ gives the value, in thousands of dollars, of an investment after $t$ years.

    a.  Identify the slope, and interpret its meaning in a complete sentence.

    b.  Identify the vertical intercept.  Write it as an ordered pair and interpret its meaning in a complete sentence.

    c.  Determine the horizontal intercept.  Write it as an ordered pair and discuss its meaning.

36. When a new charter school opened in 2005, there were 300 students enrolled.  Write a formula for the function N($t$) representing the number of students attending this charter school $t$ years after 2005, assuming that the student population

   a.  Increases by 20 students per year.

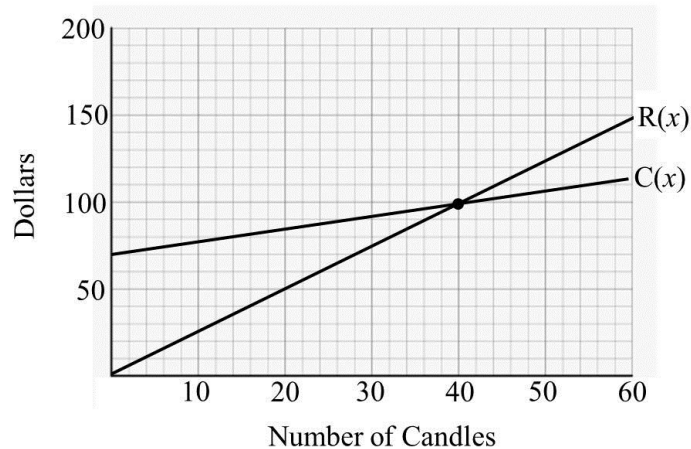   b.  Decreases by 40 students per year.

   c.  Increases by 100 students every 4 years.

   d.  Decreases by 60 students every two years.

   e.  Remains constant (does not change).

| Extension |
|:---:|

37. The graph below shows the cost and revenue for a candle company.  The function R(x) gives the revenue earned when x candles are sold.  The function C(x) gives the total cost to produce x candles.



a.  Determine the formula for C(x):  C(x) = _____

b.  Determine the formula for R(x):  R(x) = _____

c.  Profit is found by subtracting the costs from the revenue.  Determine the formula for the profit, P(x), earned from selling x candles.

d.  Identify the vertical intercept of P(x).  Write it as an ordered pair, and interpret its meaning.

e.  Identify the slope of P(x).  Interpret its meaning.

f.  Discuss the cost, revenue, and *profit* for this company when 40 candles are sold.

# Unit 10:  Review

**1.** Draw an **accurate** graph of the function $y = 3 - 5x$.
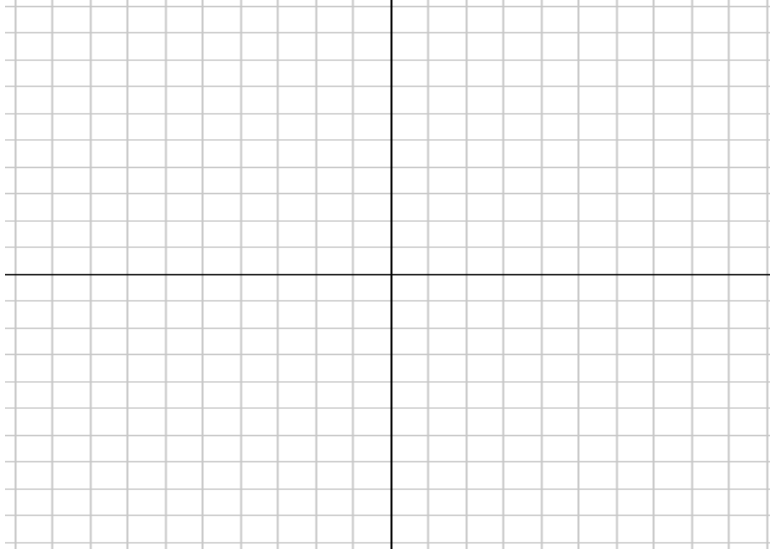
Slope:  _____

Vertical Intercept:  _____

Horizontal Intercept:  _____

2. Determine the equation of the line between the points (-6, 14) and (18, –2).  Your answer must be written in slope-intercept form.

3. Give the equation of the vertical line passing through the point (1, 8).  _____

4. Give the equation of the horizontal line passing through the point (1, 8).  _____

5. Give the equation of the linear function shown below. Write your answer in slope-intercept form.
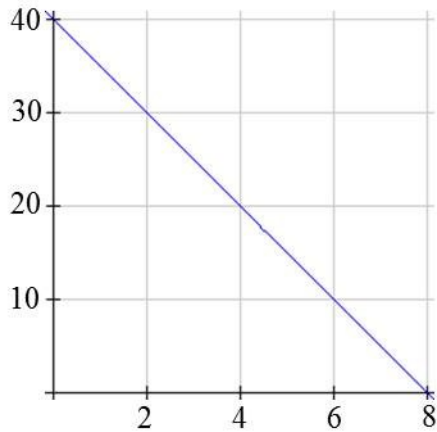


6. In the year 2000, the median cost for in-state tuition and fees at a public 4-year college was $3412. In the year 2010, the median cost for tuition had risen to $7231.

   a. Determine a linear function, $C(t)$ to represent the cost for tuition and fees $t$ years since 2000. Show all of your work. Write your answer in function notation, $C(t) = mt + b$.

   b. Determine $C(13)$. Show all of your work. Write your answer in a complete sentence.

   c. Identify the slope of this linear function and write a sentence explaining its meaning in this situation.