Unit 9: Introduction to Linear Functions

Section 9.1: Linear Functions

Section 9.2: Graphing Linear Functions

Section 9.3: Interpreting the Slope of a Linear Function

Section 9.4: Using Rates of Change to Build Tables and Graphs

Section 9.5: Is the Function Linear?

KEY TERMS AND CONCEPTS	
Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms *in your own words*. Provide examples that are not identical to those in the Media Lesson.	
Linear Functions	
Slope	
Using Slope to Graph a Linear Function	
Units of Slope	

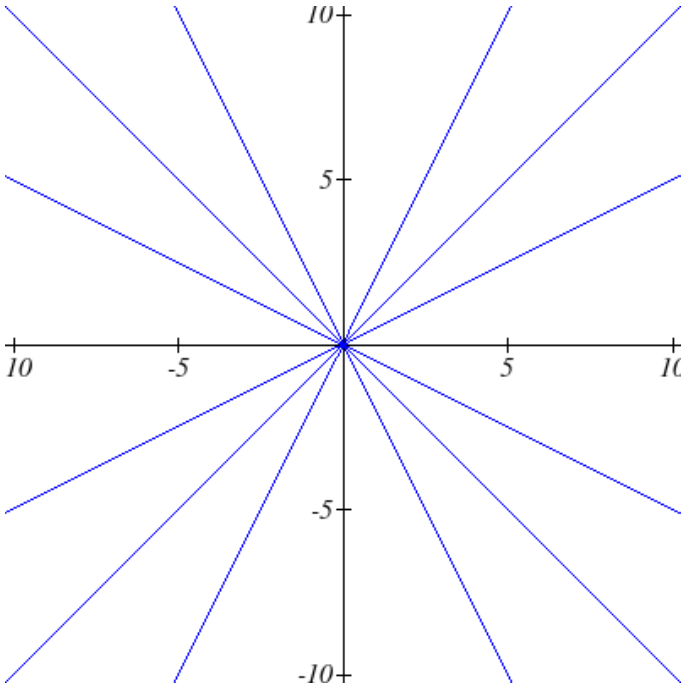
Rate of Change	
Constant Rate of Change	
Interpreting the Slope of a Linear Function	

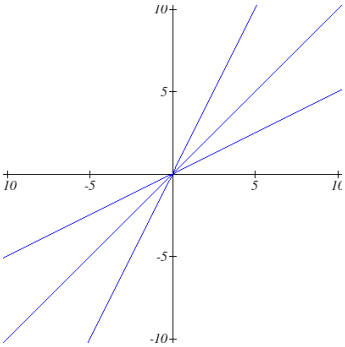
Unit 9: Media Lesson

Section 9.1: Linear Functions

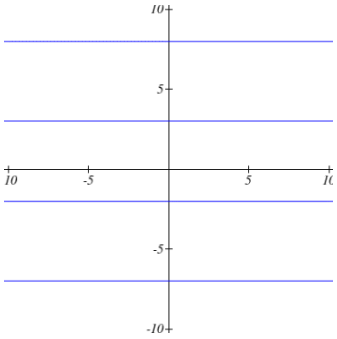
A linear function is a function that fits the form:

A linear function can be graphically represented by a _____

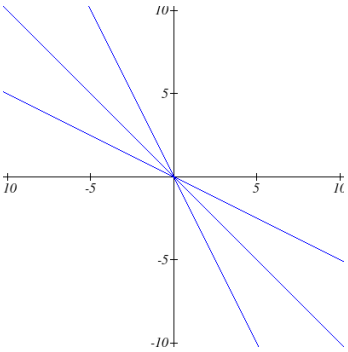




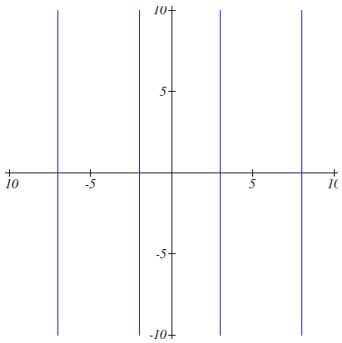
Increasing Linear Function
Slope > 0



Constant Function
Slope = 0

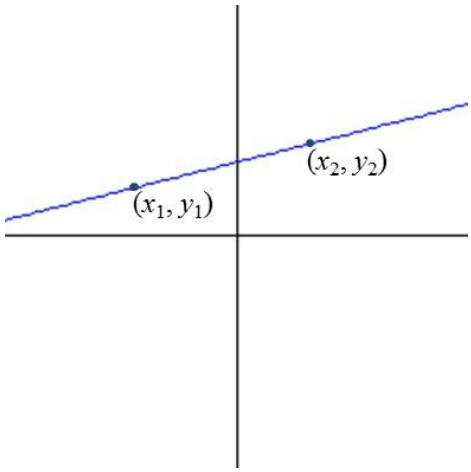


Decreasing Linear Function
Slope < 0



Not a Function
Slope is Undefined (No Slope)

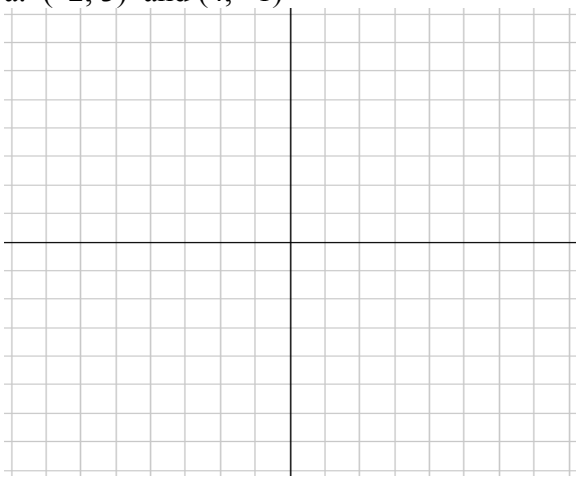
$$m = \text{Slope} = \frac{\text{Change in OUTPUT}}{\text{Change in INPUT}} = \frac{\Delta \text{OUTPUT}}{\Delta \text{INPUT}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



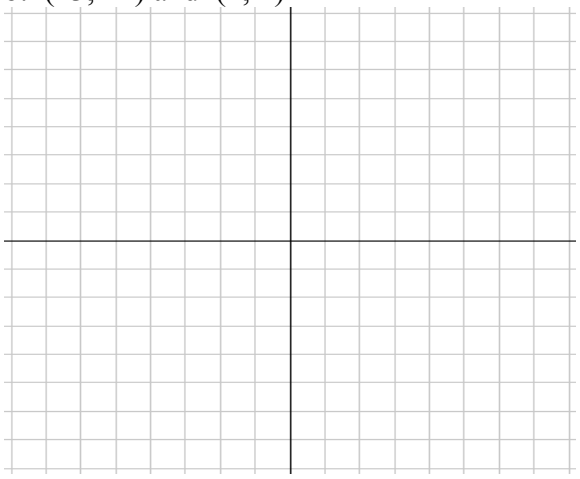
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1: Determine the slope for each of the following:

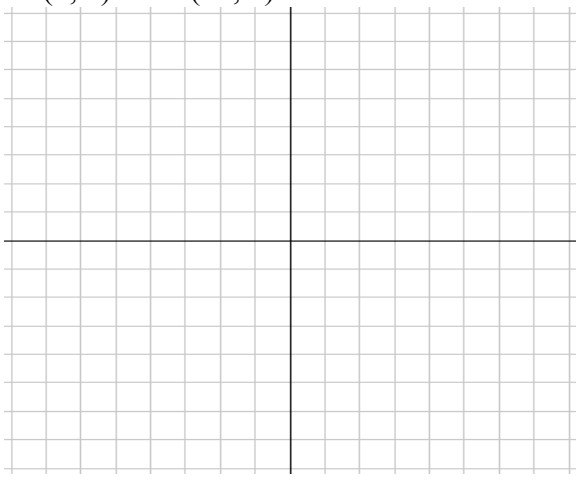
a. (–2, 3) and (4, –1)



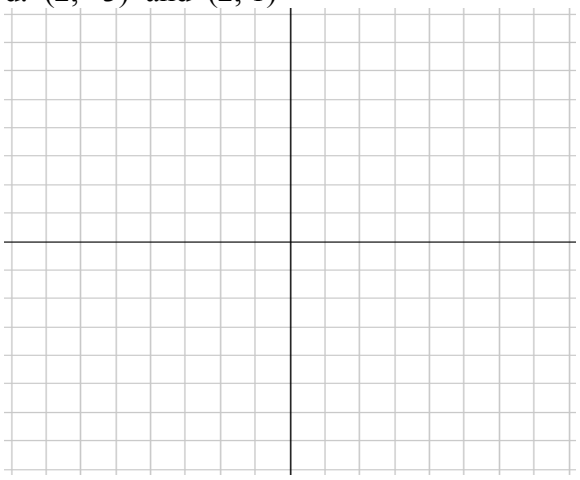
b. (–3, –1) and (4, 2)



c. (3, 2) and (–1, 2)

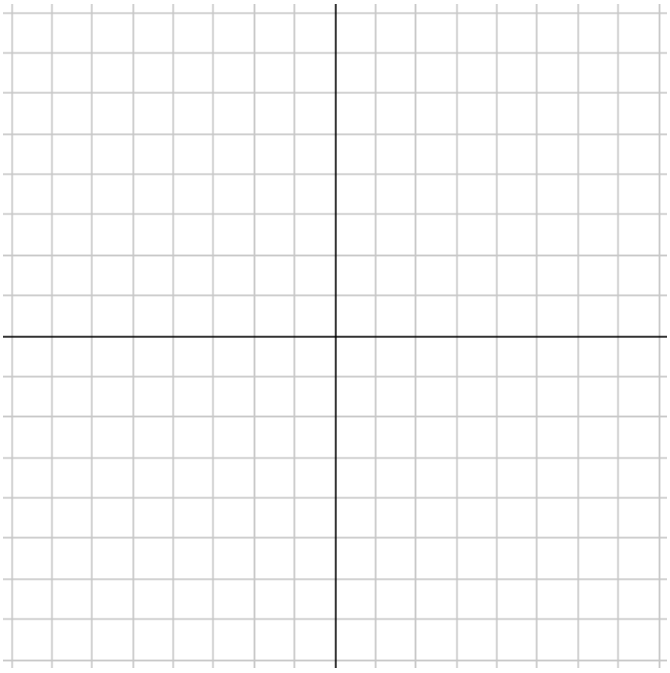


d. (2, –3) and (2, 1)



Section 9.1 – You Try

Plot the points (–4, –1) and (5, –6) and draw a line connecting them. Determine the slope of this line. Show all steps, as in the media examples.



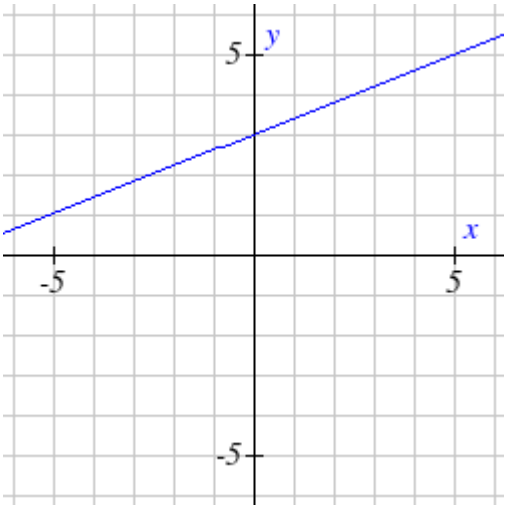
Section 9.2: Graphing Linear Functions

USING THE SLOPE TO GRAPH A LINEAR FUNCTION

$$m = \text{Slope} = \frac{\text{Change in OUTPUT}}{\text{Change in INPUT}} = \frac{\text{Vertical Change}}{\text{Horizontal Change}} \rightarrow \frac{\updownarrow}{\leftrightarrow}$$

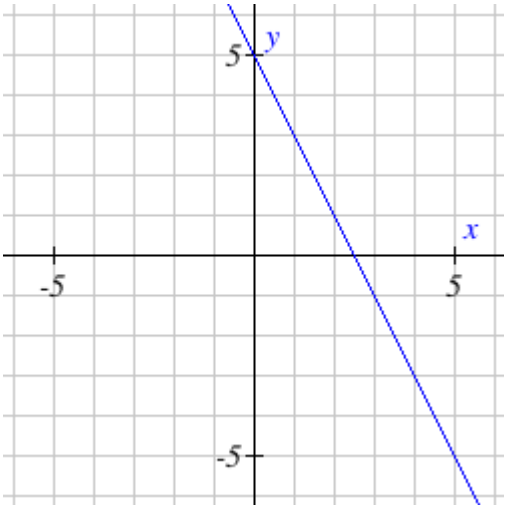
$$m = \frac{2}{5} \rightarrow \frac{\text{up } 2}{\text{right } 5}$$

$$m = \frac{2}{5} = \frac{-2}{-5} \rightarrow \frac{\text{down } 2}{\text{left } 5}$$



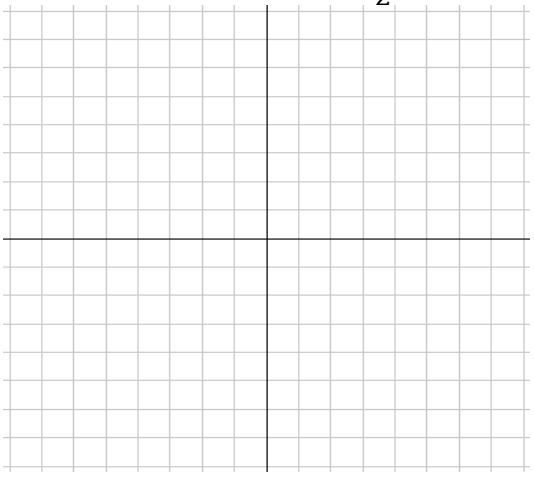
$$m = -2 = -\frac{2}{1} = \frac{-2}{1} \rightarrow \frac{\text{down } 2}{\text{right } 1}$$

$$m = -2 = -\frac{2}{1} = \frac{2}{-1} \rightarrow \frac{\text{up } 2}{\text{left } 5}$$

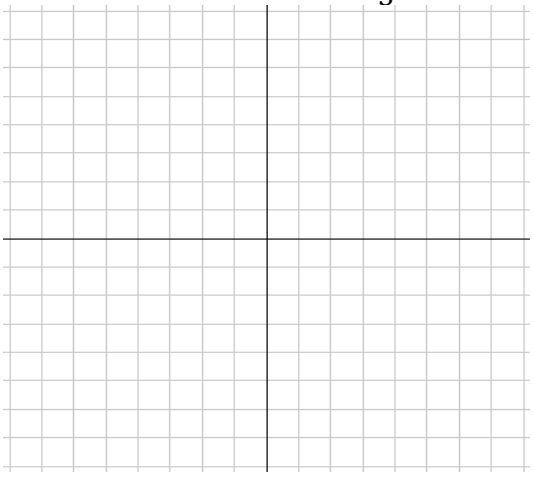


Example 1: Draw an accurate graph for each of the following

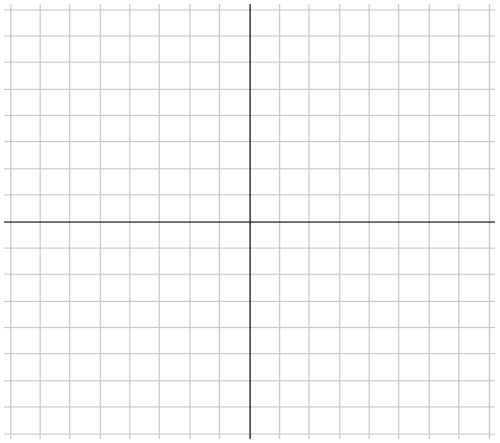
a. (–2, –3) slope $\frac{1}{2}$



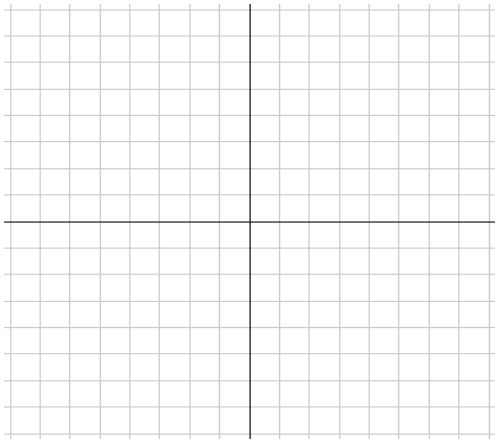
b. (0, –1) slope $-\frac{2}{3}$



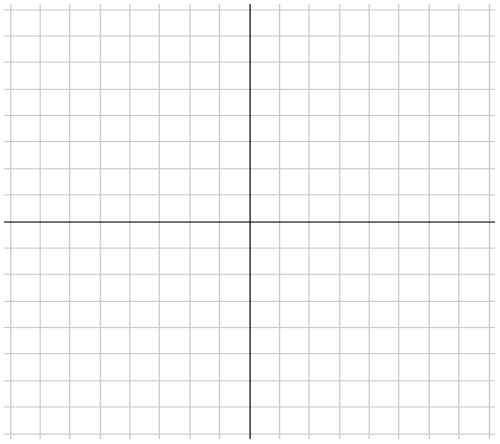
c. (2, 1) slope 3



d. (1, –4) slope 0

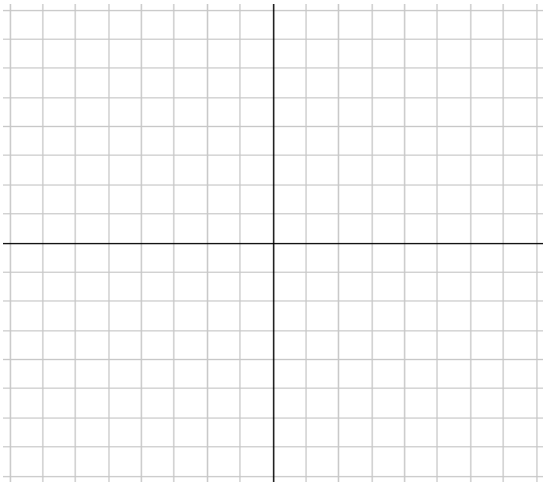


e. (5, 2) undefined slope



Section 9.2 – You Try

Sketch the graph of a linear function that passes through the point (1, –2) with slope = $-\frac{3}{5}$.



Your line must extend accurately from edge to edge of the graph shown

Give the **coordinates** of at least two additional points on the line.

_____

Section 9.3: Interpreting the Slope of a Linear Function

$$\text{Slope} = \frac{\text{Change in Output}}{\text{Change in Input}} \qquad \text{Units of Slope} = \frac{\text{Output Units}}{\text{Input Unit}} \rightarrow \text{Rate of Change}$$

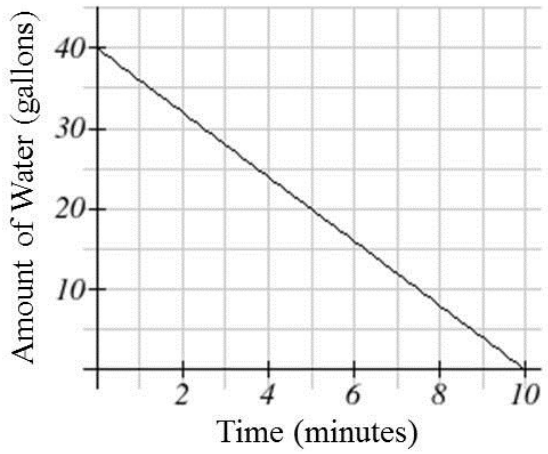
Example: Output = Height in Feet Input = Time in Seconds

$$\text{Slope} = \frac{\text{Change in Height}}{\text{Change in Time}} \qquad \text{Units of Slope} = \frac{\text{feet}}{\text{second}} = \text{feet/second}$$

What is the meaning of a slope of –5?

What is the meaning of a slope of 8?

Example 1: Consider the graph shown below.



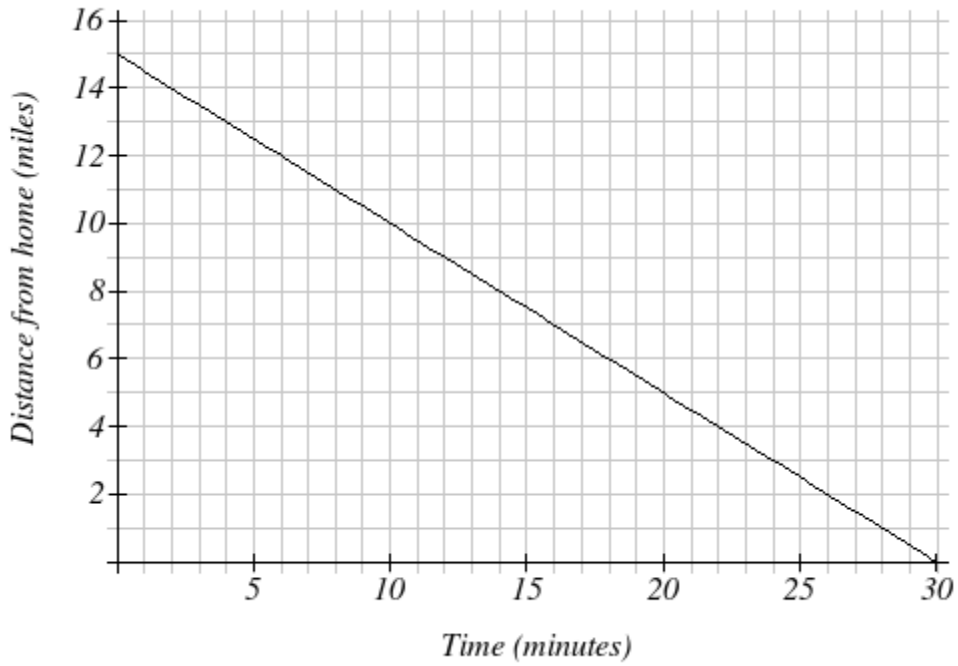
a. Identify the vertical intercept and interpret its meaning.

b. Identify the horizontal intercept and interpret its meaning.

c. Determine the slope, and interpret its meaning.

Section 9.3 – You Try

The graph below shows Sally's distance from home over a 30 minute time period.



a. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning.

b. Identify the horizontal intercept. Write it as an ordered pair and interpret its meaning.

c. Determine the slope, and interpret its meaning. Show your work and write your answer in a complete sentence.

Section 9.4: Using Rates of Change to Build Tables and Graphs

For each of the examples below, *circle* the rate of change in each situation and *underline* the starting value. Then use the given information to complete the table. Graph the results, and decide if it would make sense to connect the data points on the graph.

Example 1: A local carpet cleaning company charges $15 for each room plus a nonrefundable reservation fee of $25.

Number of Rooms	*Total Cost (dollars)*
0	
1	
2	
3	
4	
5	
6	

Example 2: Water is leaking out of a tank at a constant rate of 2 gallons per minute. The tank initially held 12 gallons of water.

Time (minutes)	*Amount of Water in Tank (gallons)*
0	
1	
2	
3	
4	
5	
6	

Section 9.4 – You Try

Sara is selling snow cones at the local carnival for $3 each.

Identify the rate of change in this situation. Be sure to include units in your answer.

Complete the table to show Sara's revenue from selling the snow cones. Graph the results, and decide if it would make sense to connect the data points on the graph.

Number of Snow Cones	Revenue (in dollars)
0	
1	
2	
3	
4	
5	
6	

Section 9.5: Is the Function Linear?

Rate of Change of a Linear Function
Given any two points (x_1, y_1) and (x_2, y_2), the **rate of change** between the points on the interval x_1 to x_2 is determined by computing the following ratio: $$\text{Rate of Change} = \frac{\text{Change in Output}}{\text{Change in Input}} = \frac{y_2 - y_1}{x_2 - x_1}$$ If the function is LINEAR, then the rate of change will be *the same* between any pair of points. This constant rate of change is the SLOPE of the linear function.

Example 1: Determine if the following function is linear by computing the rate of change between several pairs of points. If it is linear, give the slope.

x	y
-5	23
-2	14
0	8
3	-1
8	-16

Example 2: Determine if the following function is linear by computing the rate of change between several pairs of points. If it is linear, give the slope.

n	$T(n)$
-6	-3
-2	-1
0	1
1	2
4	6

Example 3: Determine if the following function is linear by computing the rate of change between several pairs of points. If it is linear, give the slope.

x	$g(x)$
-5	3
-2	3
0	3
4	3
6	3

Section 9.5 – You Try

Determine if the following function is linear by computing the rate of change between several pairs of points. If it is linear, give the slope. Show all of your work, as in the media examples.

x	y
-8	-30
-3	-10
0	2
2	10
5	22

Unit 9: Practice Problems

Skills Practice

1. Determine the slope of the line between each of the following pairs of points. Show all steps, and reduce your answer to lowest terms.

a. (4, –5) and (–2, 3)

b. (–3, 2) and (1, 8)

c. (5, –9) and (5, 2)

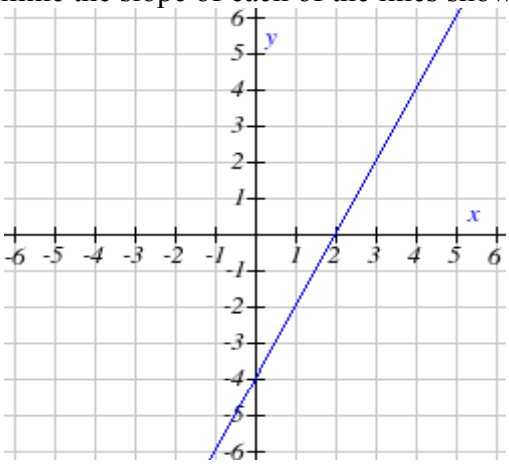
d. (2, –1) and (–2, 3)

e. (4, 3) and (12, –3)

f. (2, –4) and (7, –4)

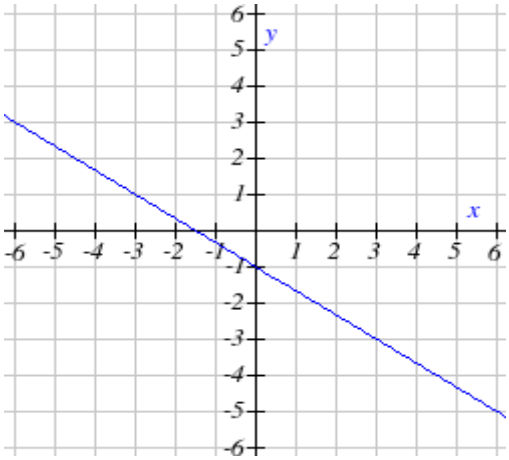
2. Determine the slope of each of the lines shown below.

a.



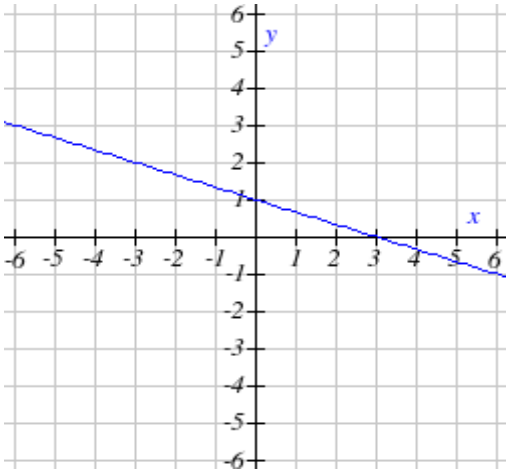
Slope = _____

b.



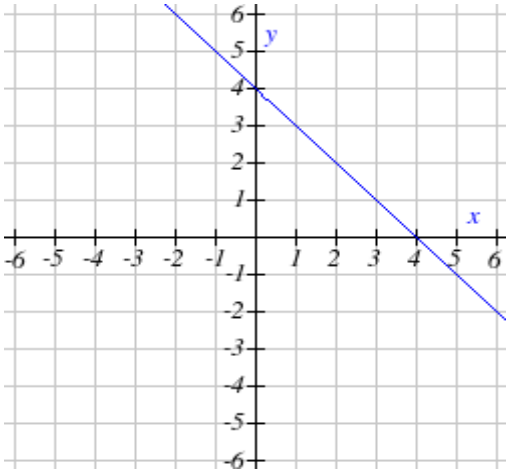
Slope = _____

c.



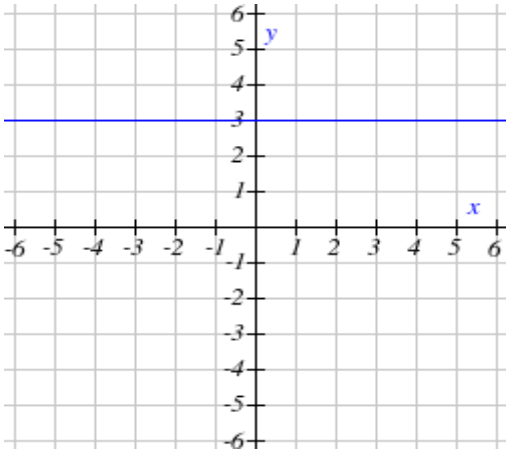
Slope = _____

d.



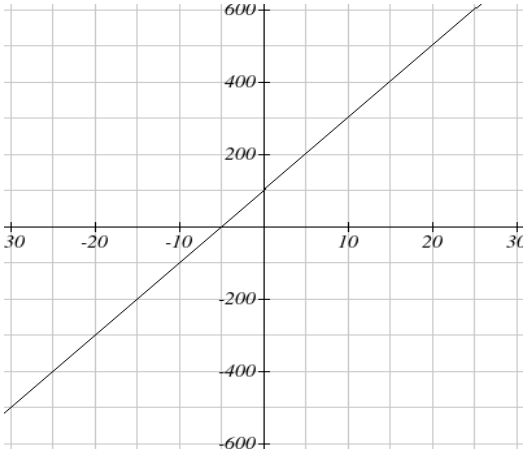
Slope = _____

e.



Slope = _____

f.

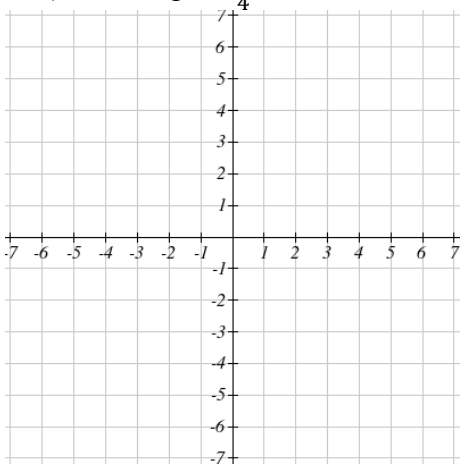


Slope = _____

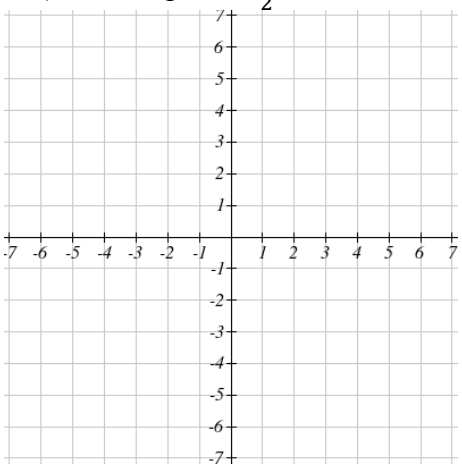
3. Draw an **accurate** graph for each of the following by

- Plotting the point
- Using the slope to find at least two additional points

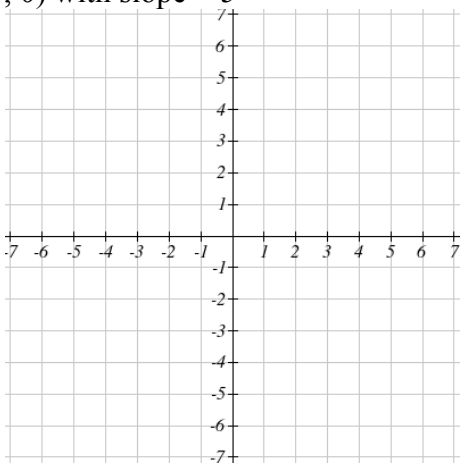
a. (1, –2) with slope = $\frac{1}{4}$



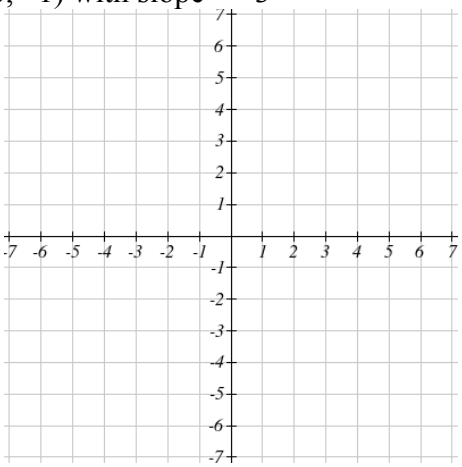
b. (–1,3) with slope = $-\frac{3}{2}$



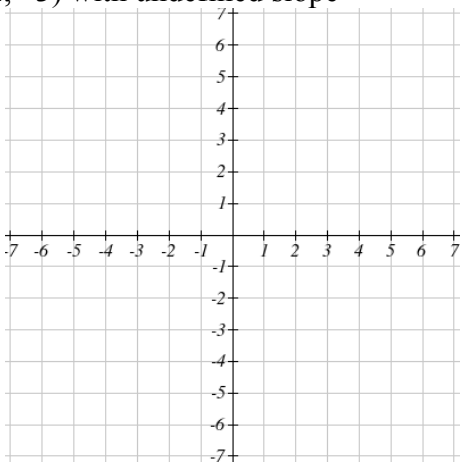
c. (3, 0) with slope = 5



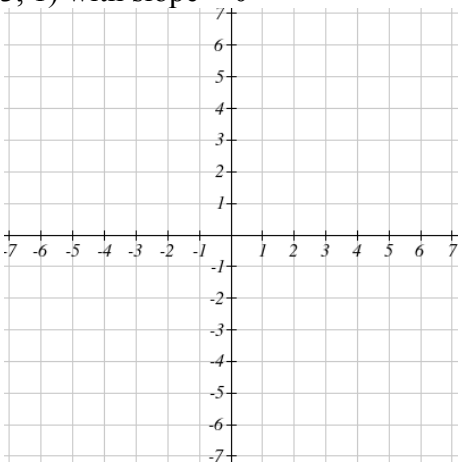
d. (0, –1) with slope = –3



e. (2, –3) with undefined slope



f. (–3, 1) with slope = 0



4. For each of the following, determine if the function is linear by computing the rate of change between several pairs of points. If it is linear, give the slope.

a.

x	y
-3	2
-1	8
0	16
2	64
3	128

b.

n	$A(n)$
-4	28
-1	19
5	1
11	-17
14	-26

c.

t	$r(t)$
-6	5
-3	6
4	7
11	8
18	9

Applications

5. A candy company has a machine that produces candy canes. The number of candy canes produced depends on the amount of time the machine has been operating. The machine produces 160 candy canes in five minutes. In twenty minutes, the machine can produce 640 candy canes. Determine the rate of change in this situation, and write a sentence explaining its meaning.

6. The enrollment at a local charter has been decreasing linearly. In 2006, there were 857 students enrolled. By 2015, there were only 785 students enrolled. Determine the rate of change of this school's enrollment during this time period.

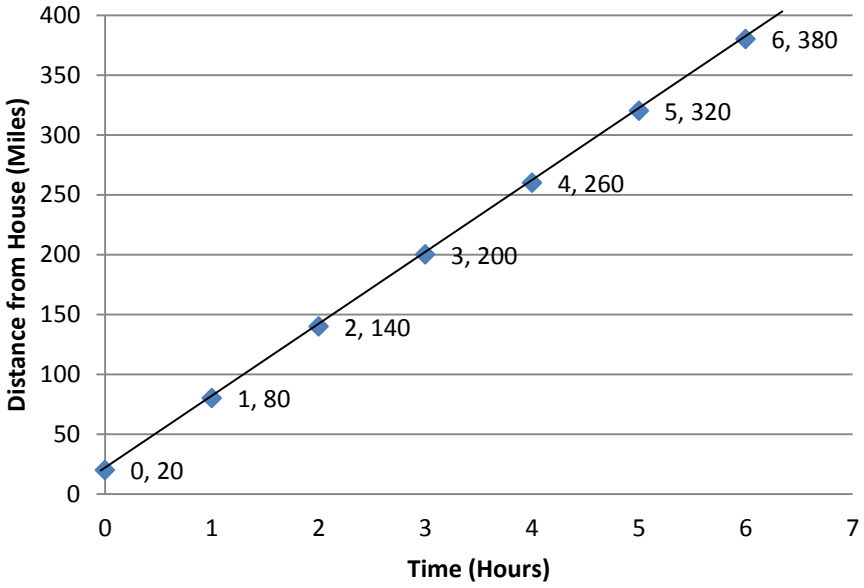
7. A tree grows 2 feet taller every 3 years. Determine the rate of change in this situation.

8. Oil is leaking from a tanker at a rate of 18 gallons every 30 minutes. Determine the rate of change in this situation.

9. In the year 1987, an investment was worth $30,200. In the year 1996, this investment was worth $43,700. Determine the rate of change in this situation, and write a sentence explaining its meaning.

10. In the year 1998, the surface elevation of Lake Powell was 3,843 feet above sea level. In the year 2001, the surface elevation of Lake Powell was 3,609 feet above sea level. Determine the rate of change in this situation, and write a sentence explaining its meaning.

11. The graph below shows the distance you are from your house if you leave work and drive in the opposite direction.



a. In a complete sentence, interpret the ordered pair (2, 140)

b. Identify the vertical intercept and interpret its meaning.

c. Determine the slope, and interpret its meaning.

d. At this rate, how far away from home will you be after 7 hours?

e. At this rate, how long will it take for you to be 680 miles from your home?

12. You need to hire a caterer for a banquet.

a. Caterer A charges a nonrefundable delivery fee of $45 plus $5 per guest.

b. Caterer B charges a fee of $150. This includes the delivery and food for up to 30 guests.

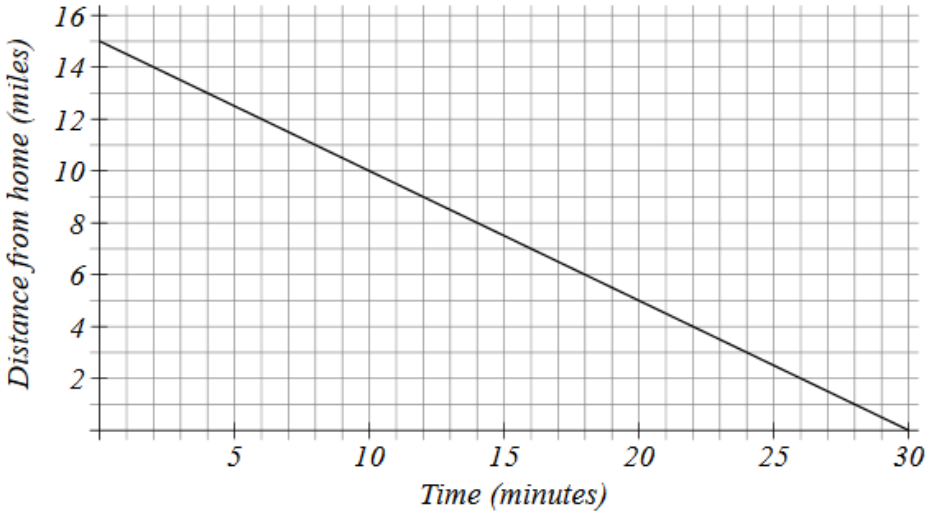
Use this information to complete the tables below. Draw good graphs of your results.

Number of Guests	*Cost (dollars) Caterer A*
0	
1	
2	
3	
4	
5	
6	

Number of Guests	*Cost (dollars) Caterer B*
0	
1	
2	
3	
4	
5	
6	

Which caterer should you choose? What considerations should be made before making this decision?

13. The function D(t) below shows Sally's distance from home over a 30 minute time period.



a. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning.

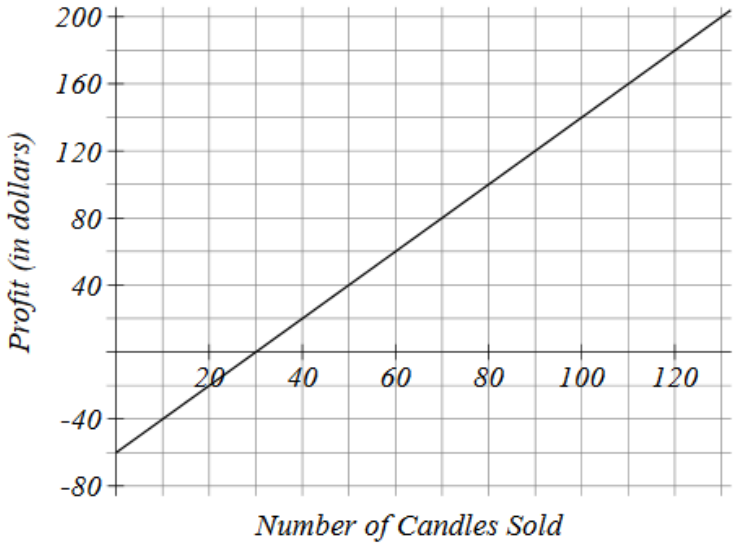
b. Identify the horizontal intercept. Write it as an ordered pair and interpret its meaning.

c. Determine the slope of D(t), and interpret its meaning.

d. Determine the practical domain of this linear function. Use inequality notation and include units.

e. Determine the practical range of this linear function. Use inequality notation and include units.

14. Janey is selling homemade scented candles. The graph below shows her profit from selling the candles.



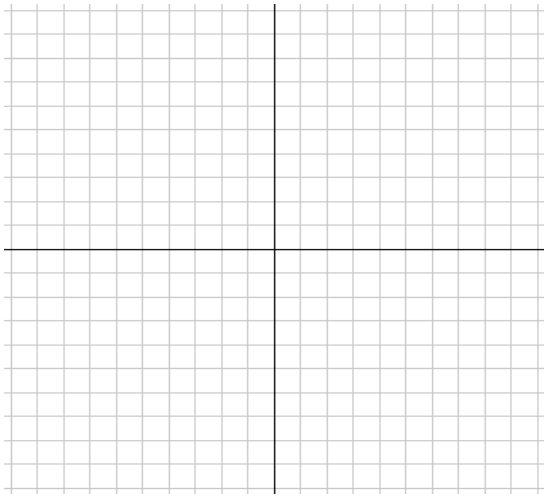
a. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning.

b. Identify the horizontal intercept. Write it as an ordered pair and interpret its meaning.

c. Determine the slope, and interpret its meaning.

Extension

15. Graph the lines A, B, C, and D on the grid below.



A: Passes through the point (0, -5) with slope $\frac{2}{3}$

B: Passes through the point (0, -1) with slope $\frac{2}{3}$

C: Passes through the point (0, 3) with slope $\frac{2}{3}$

D: Passes through the point (0, 7) with slope $\frac{2}{3}$

How are these lines geometrically related?

16. Amber starts off with $1000 in her savings account. Determine the balance in the account after 1 year in each of the following situations:

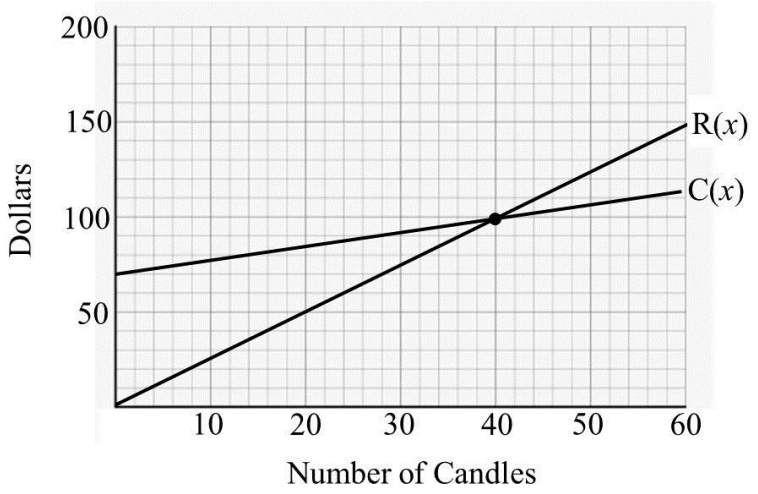
a. Amber deposits $50 every month.

b. Amber withdraws $50 from her account every month.

c. Amber deposits $500 into the account every six months.

d. Amber makes no withdrawals or deposits.

17. The graph below shows the cost and revenue for a company that produces and sells scented candles. The function R(x) gives the revenue earned when x candles are sold. The function C(x) gives the total cost to produce x candles.



a. Identify the vertical intercept of C(x). Write it as an ordered pair, and interpret its meaning.

b. Determine the slope of C(x). Interpret its meaning.

c. Identify the vertical intercept of R(x). Write it as an ordered pair, and interpret its meaning.

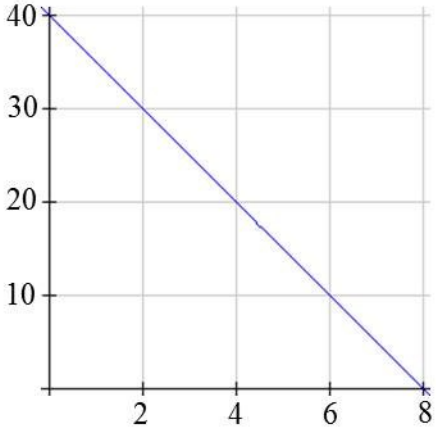
d. Determine the slope of R(x). Interpret its meaning.

e. Discuss the significance of the point (40, 100) in terms of the cost, revenue, and *profit* for this company.

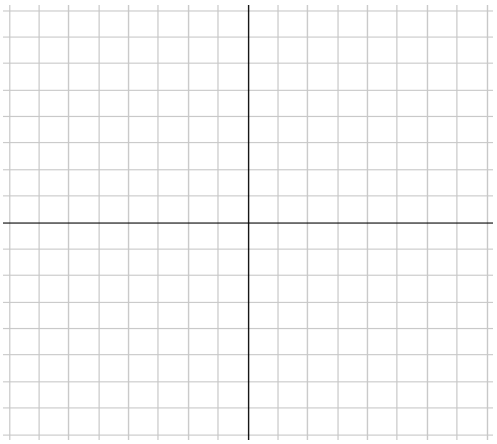
Unit 9: Review

1. Determine the slope of the line between the points (2, –1) and (–2, 3). Show all steps, and reduce your answer to lowest terms.

2. Determine the slope of the line shown below.



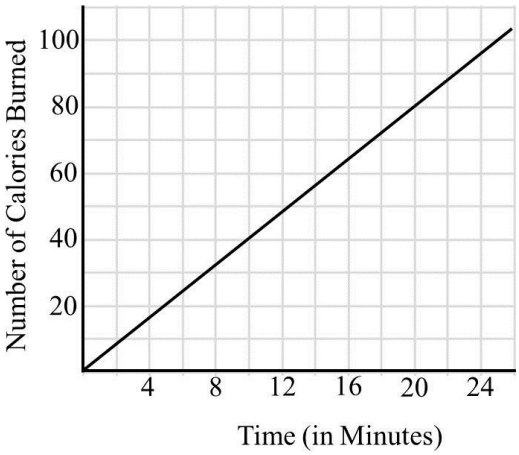
3. Draw an **accurate** graph of the line passing through the point (–2,4) with slope $-\frac{2}{5}$.



4. Determine if the function $g(x)$ is linear by computing the rate of change between several pairs of points. If it is linear, give the slope.

x	$g(x)$
-8	39
-2	18
0	11
4	-3
12	-31

5. The graph of the function $C(n)$ below shows the number of calories burned after riding a stationary bike for n minutes.



a. Interpret the meaning of the statement $C(8) = 32$

b. Determine $C(10)$ and interpret its meaning in a complete sentence.

c. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

d. Determine the slope of $C(n)$ and interpret its meaning in a complete sentence.