Unit 4: Inequalities

Section 4.1: Linear Inequalities

Section 4.2: Solving Linear Inequalities

Section 4.3: Solving Inequalities – Applications

Section 4.4: Compound Inequalities

Section 4.5: Absolute Value Equations and Inequalities

KEY TERMS AND CONCEPTS	
Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms *in your own words*. Provide examples that are not identical to those in the Media Lesson.	
Inequality Symbols	
Algebraic Inequality	
Compound Inequality	
Interval Notation	

Solution to an Algebraic Inequality	
Solution Set	
Absolute Value	
Absolute Value Equation	
Absolute Value Inequality	

Unit 4: Media Lesson

Section 4.1: Inequalities

Symbol	In words	Examples
$<$		
$\leq$		
$>$		
$\geq$		
$\neq$		

Definitions
An **algebraic inequality** is a mathematical sentence connecting an expression to a value, variable, or another expression with an inequality sign.
A **solution** to an inequality is a value that makes the inequality true.

Example 1: Determine whether the number 4 is a solution to the following inequalities.
$$x > 1 \qquad x < 1 \qquad x \leq 9 \qquad x > 4 \qquad x \geq 4$$

THE SOLUTION SET OF A LINEAR INEQUALITY

Inequality	Graph	Interval Notation
$x > 2$	$-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$ $-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$	
$x \geq 2$	$-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$ $-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$	
$x < 2$	$-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$ $-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$	
$x \leq 2$	$-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$ $-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$	

Translate a statement into an inequality

Example 2: Write an inequality to represent the following situation. Clearly indicate what the variable represents.

    a. In order to go on the ride, a child must be more than 48 inches tall.

    b. Jordan can spend at most $10 on lunch.

Section 4.1 – You Try

Complete the following problems.

    a. Which of the following values are in the solution set for $n < 5$?

$$n = -3 \qquad n = 0 \qquad n = 4.99 \qquad n = 5 \qquad n = 12$$

    b. Translate the statement into an inequality.   Let a represent the age of a child.

$$\text{Children age 2 and under are free at Disneyland}$$

    c. Complete the table below:

Inequality	Graph	Interval Notation
$x \geq -3$	$-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$	
	$-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$	$(-\infty, 11]$
	$-\infty \longleftarrow\!\!\blacktriangleleft\!\!\underset{\text{-5 -4 -3 -2 -1 0 1 2 3 4 5}}{\longrightarrow}\!\!)\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$	

Section 4.2: Solving Linear Inequalities

| STEPS FOR SOLVING A LINEAR INEQUALITY |
|---|
| 1. Simplify each side of the inequality. Remove parenthesis if necessary. Collect like terms. |
| 2. Add or subtract terms on each side of the inequality so that all terms containing the variable are on one side and all constant terms are on the other side. |
| 3. Simplify each side of the inequality by combining like terms. |
| *4. Multiply or divide on both sides to isolate the variable. CAUTION!!! If you multiply or divide both sides of an inequality by a **negative number**, you have to reverse the inequality sign.* |
| 5. Check by substituting the solution (*endpoint and a value from the solution set*) into the original inequality. |

Solve the inequality, check your answer, and graph the solution on a number line.

Example 1: Solve the inequality, check your answer, and graph the solution on a number line.

$3x > x + 6$ Graph:

$$-\infty \longleftarrow \hspace{3cm} \longrightarrow \infty$$
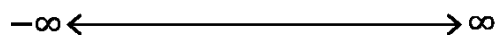
Interval Notation: _____

Example 2: Solve the inequality and graph the solution on a number line.

$3 - 5a \le 2(a + 5)$ Graph:

$$-\infty \longleftarrow \hspace{3cm} \longrightarrow \infty$$
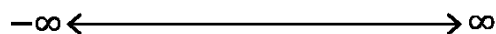
Interval Notation: _____

Example 3: Solve the inequality and graph the solution on a number line.

$-5(x + 2) \geq -3(x + 4)$ Graph:

$-\infty \longleftarrow \xrightarrow{\hspace{4cm}} \infty$

Interval Notation: _____

Section 4.2 – You Try

Solve the inequality, check your answer, and graph the solution on a number line. Show all steps as in the media examples.

a. $7 - 4x \geq -5$ Graph:

$-\infty \longleftarrow \xrightarrow{\hspace{4cm}} \infty$

Interval Notation: _____

.

b. $6x + 13 < 5(2x - 3)$ Graph:

$-\infty \longleftarrow \xrightarrow{\hspace{4cm}} \infty$

Interval Notation: _____

Section 4.3: Solving Inequalities – Applications

> For each problem, underline the Givens and circle the Goal. Form a Strategy, Solve, and Check. Write your answer in a complete sentence.

Example 1: The cost of tuition is $76 per credit hour. Write an *inequality* that can be used to determine the number of credit hours a student can take for under $1000. Solve the inequality, and write your answer in a complete sentence.

Example 2: Sean owns a business that builds computers. The fixed operating costs for his business are $2,700 per week. In addition to fixed operating costs, each computer costs $600 to produce. Each computer sells for $1,500.   Write an *inequality* that can be used to determine the number of computers Sean needs to sell in order make a profit each week. Solve the inequality, and write your answer in a complete sentence.

Section 4.3 – YOU TRY

Gasoline costs $3.79 per gallon.

    a. Write an *inequality* that can be used to determine how many gallons of fuel can be purchased for under $20.   Clearly indicate what the variable represents.

    b. Solve the inequality in part a, and write your answer in a complete sentence. Show all steps as in the media examples.

Section 4.4: Compound Inequalities

THE SOLUTION SET OF A COMPOUND INEQUALITY		
Inequality	**Graph**	**Interval Notation**
$-1 < x < 2$	$-\infty \longleftarrow\!\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$ $-\infty \longleftarrow\!\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$	
$-1 \le x \le 2$	$-\infty \longleftarrow\!\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$ $-\infty \longleftarrow\!\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$	
$-1 \le x < 2$	$-\infty \longleftarrow\!\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$ $-\infty \longleftarrow\!\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$	

Verify that a given value is a solution to the inequality

Example 1: Which of the following values are in the solution set for $-3 \le n < 5$?

$n = -5 \qquad n = -3 \qquad n = 0 \qquad n = 4.99 \qquad n = 5 \qquad n = 12$

$$-\infty \longleftarrow\!\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$$

Translate a statement into an inequality

Example 2: Write a compound inequality to represent the following situation. Clearly indicate what the variable represents.

 a. A number is greater than or equal to 5 but less than 8.

 b. My car's tank can hold a maximum of 20 gallons of gas.

Section 4.4 – You Try

Complete the following problems.

a. Which of the following values are in the solution set for $-8 \le w < 2$? Show your work.

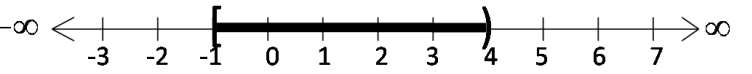$w = -11$ $w = -8$ $w = -5$ $w = 0$ $w = 2$ $w = -8.5$

b. Translate the statement into a compound inequality.

A number, n, is greater than 0, and less than or equal to 8.

c. Complete the table below:

Inequality	Graph	Interval Notation
$0 < x < 4$	$-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$	
	$-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$	$(-3, 1]$
	$-\infty \longleftarrow\!\!+\!\!+\![\!\rule{3cm}{1pt}\!)\!+\!+\!+\!\longrightarrow \infty$ -3 -2 -1 0 1 2 3 4 5 6 7	

Section 4.5: Absolute Value Equations and Inequalities

Absolute Value

$$-\infty \; \longleftarrow \; | \quad | \quad | \quad | \quad | \quad | \quad | \quad | \quad | \quad | \quad | \; \longrightarrow \; \infty$$

Example 1: Evaluate the following: $|2| =$ $|-2| =$

Absolute Value Equations

Determine the solution to each of the following equations.

Example 2: $|x| = 2$ $|x| = 3$ $|x| = -4$

Absolute Value Inequalities	
$\|x\| < 2$	List some values in the solution set: _____ $-\infty \; \longleftarrow \; \| \quad \| \quad \| \quad \| \quad \| \quad \| \quad \| \quad \| \quad \| \quad \| \quad \| \; \longrightarrow \; \infty$
$\|x\| \leq 2$	List some values in the solution set: _____ $-\infty \; \longleftarrow \; \| \quad \| \quad \| \quad \| \quad \| \quad \| \quad \| \quad \| \quad \| \quad \| \quad \| \; \longrightarrow \; \infty$
$\|x\| > 2$	List some values in the solution set: _____ $-\infty \; \longleftarrow \; \| \quad \| \quad \| \quad \| \quad \| \quad \| \quad \| \quad \| \quad \| \quad \| \quad \| \; \longrightarrow \; \infty$

Section 4.5 – You Try

Complete the following problems.

a. Determine the solution to the equation $|x| = 8$

b. Absolute Value Inequalities:

$|x| < 3$	List some values in the solution set: _____ $-\infty \longleftarrow\!\!+\!\!\!\!+\!\!\!\!+\!\!\!\!+\!\!\!\!+\!\!\!\!+\!\!\!\!+\!\!\!\!+\!\!\!\!+\!\!\!\!+\!\!\!\!+\!\!\longrightarrow \infty$
$|x| \geq 3$	List some values in the solution set: _____ $-\infty \longleftarrow\!\!+\!\!\!\!+\!\!\!\!+\!\!\!\!+\!\!\!\!+\!\!\!\!+\!\!\!\!+\!\!\!\!+\!\!\!\!+\!\!\!\!+\!\!\!\!+\!\!\longrightarrow \infty$

Unit 4: Practice Problems

1. For each of the following, circle *all* correct answers.

   a. Which of the given values are in the solution set for $x < 3$?

   $$x = 0 \qquad x = -1 \qquad x = -5 \qquad x = 3 \qquad x = 5 \qquad x = -\frac{5}{3}$$

   b. Which of the given values are in the solution set for $x \geq -1$?

   $$x = 0 \qquad x = -1 \qquad x = -5 \qquad x = 3 \qquad x = 5 \qquad x = -\frac{5}{3}$$

   c. Which of the given values are in the interval $[-2, \infty)$?

   $$x = 0 \qquad x = -1 \qquad x = -5 \qquad x = 3 \qquad x = 5 \qquad x = -\frac{5}{3}$$

   d. Which of the given values are in the interval $(-\infty, -1)$?

   $$x = 0 \qquad x = -1 \qquad x = -5 \qquad x = 3 \qquad x = 5 \qquad x = -\frac{5}{3}$$

   e. Which of the given values are in the interval $(-1, 5]$?

   $$x = 0 \qquad x = -1 \qquad x = -5 \qquad x = 3 \qquad x = 5 \qquad x = -\frac{5}{3}$$

   f. Which of the given values are in the interval $-5 < x \leq 3$?

   $$x = 0 \qquad x = -1 \qquad x = -5 \qquad x = 3 \qquad x = 5 \qquad x = -\frac{5}{3}$$

2. Complete the table below:

Inequality	Graph	Interval Notation
$x > 8$	$-\infty \longleftrightarrow \infty$	
$x \leq -1$	$-\infty \longleftrightarrow \infty$	
$8 \leq x < 12$	$-\infty \longleftrightarrow \infty$	
	$-\infty \longleftrightarrow \infty$	$(-2, \infty)$
	$-\infty \longleftrightarrow \infty$	$(-\infty, 6]$
	$-\infty \longleftrightarrow \infty$	$(-10, -7]$

3. Solve the inequality, showing all steps. Write your answer as an inequality *and* in interval notation, then graph the solution set on the number line.

$4x \leq 2x + 12$ Interval Notation: _____

Graph:

$-\infty \longleftarrow \longrightarrow \infty$

4. Solve the inequality, showing all steps. Write your answer as an inequality *and* in interval notation, then graph the solution set on the number line.

$14m + 8 > 6m - 8$ Interval Notation: _____

Graph:

$-\infty \longleftarrow \longrightarrow \infty$

5. Solve the inequality, showing all steps. Write your answer as an inequality *and* in interval notation, then graph the solution set on the number line.

$5(-2a - 8) \leq -9a + 4$ Interval Notation: _____

Graph:

$-\infty \longleftarrow \longrightarrow \infty$

6. For each of the following, circle *all* correct answers.

   a. Which of the given values are in the solution set for $|x| = 5$?

   $x = 0$ $x = -1$ $x = -5$ $x = -7$ $x = 3$ $x = 5$ $x = 9$

   b. Which of the given values are in the solution set for $|x| \geq 5$?

   $x = 0$ $x = -1$ $x = -5$ $x = -7$ $x = 3$ $x = 5$ $x = 9$

   c. Which of the given values are in the solution set for $|x| < 5$?

   $x = 0$ $x = -1$ $x = -5$ $x = -7$ $x = 3$ $x = 5$ $x = 9$

7. Graph the solution set for the inequalities shown below.

   a. $|x| < 1$ $-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$

   b. $|x| \geq 4$ $-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$

Applications

8.   Translate each of the given statements into an algebraic inequality.

  a. You must be at least 13 years of age in order to view a PG-13 movie. Let a represent your age.

  b. Your car's gas tank can hold up to 25 gallons of gas. Let g represent the number of gallons in your gas tank.

  c. A company must sell more than 850 items in order to make a positive profit. Let n represent the number of items sold.

9. You have $1200 for your trip to the beach. You estimate that it will cost $160 a day for food, entertainment and hotel, plus $230 round trip air fair.
  a. Write an *inequality* that can be used to determine the maximum number of days you can stay at the beach. Clearly indicate what the variable represents.

  b. Solve the inequality, and interpret your answer in a complete sentence.

10. Let p represent the marked price of an item at Toys R Us. Bella's aunt gave her a $100 gift card to Toys R Us for her birthday.
   a. If sales tax is currently 9%, set up an algebraic *inequality* to express how much she can spend using her gift card. Clearly indicate what the variable represents.

   b. Solve the inequality, and interpret your answer in a complete sentence.

11. Your car is worth $1000 at most. It is old. You find out that it needs repairs to pass inspection. The auto shop tells you that the parts cost a total of $520, and the labor cost is $68 per hour. If the repairs are more than the car is worth, you are going to donate the car to charity.
   a. Write an *inequality* that can be used to determine the maximum number of hours the mechanic can spend working on your car to help you decide to repair it or donate it. Clearly indicate what the variable represents.

   b. Solve the inequality, and interpret your answer in a complete sentence.

Extension

12. The maximum heart rate, M, is the highest heart rate achieved during maximal exercise. In general, you gain the most benefits and lessen the risks when you exercise within your *target* heart rate zone. Usually this is when your exercise heart rate is between 60 and 80 percent of your maximum heart rate. Let T represent your target heart rate. Write a compound inequality to represent this situation.
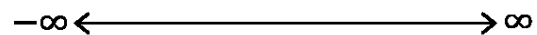
13. Solve the *compound* inequality, showing all steps. Write your answer as an inequality *and* in interval notation, then graph the solution set on the number line.

$1 < x + 3 \leq 7$

Interval Notation: _____

Graph:

$-\infty \longleftarrow \hspace{3cm} \longrightarrow \infty$

14. Solve the *compound* inequality, showing all steps. Write your answer as an inequality *and* in interval notation, then graph the solution set on the number line.

$-12 < 4n < 20$

Interval Notation: _____

Graph:

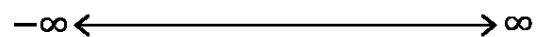$-\infty \longleftarrow \hspace{3cm} \longrightarrow \infty$

15. Solve the *compound* inequality, showing all steps. Write your answer as an inequality *and* in interval notation, then graph the solution set on the number line.

$3 \leq 2v - 5 < 11$

Interval Notation: _____

Graph:

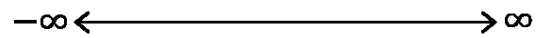$-\infty \longleftarrow \hspace{3cm} \longrightarrow \infty$

16. Solve the *compound* inequality, showing all steps. Write your answer as an inequality *and* in
    interval notation, then graph the solution set on the number line.

    $-27 \leq -3x \leq 30$

    Interval Notation: _____

    Graph:

    $-\infty \longleftarrow\!\longrightarrow \infty$

17. Solve the *compound* inequality, showing all steps. Write your answer as an inequality *and* in
    interval notation, then graph the solution set on the number line.

    $2 < 6 - 4m \leq 6$

    Interval Notation: _____

    Graph:

    $-\infty \longleftarrow\!\longrightarrow \infty$
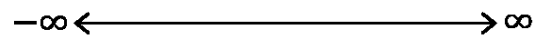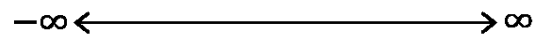
18. Solve the *compound* inequality, showing all steps. Write your answer as an inequality *and* in
    interval notation, then graph the solution set on the number line.

    $-1 \leq 5 - \frac{2}{3}b < 1$

    Interval Notation: _____

    Graph:

    $-\infty \longleftarrow\!\longrightarrow \infty$

Unit 4: Review

1. Which of the given values are in the interval $(-1, \infty)$? Circle all that apply.

   $x = 0$ $x = -1$ $x = -5$ $x = 3$

2. Which of the given values are in the interval $(-3, 5]$? Circle all that apply.

   $x = 8$ $x = -2$ $x = -3$ $x = 5$

3. You have $1400 for your trip to the beach. You estimate that it will cost $250 a day for food, entertainment and hotel, plus $198 for round trip air fair.

   a. Write an *inequality* that can be used to determine the maximum number of full days you can stay at the beach. Clearly indicate what the variable represents.
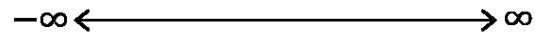
   b. Solve the inequality, and interpret your answer in a complete sentence.

4. Solve the inequality, showing all steps. Write your answer as an inequality *and* in interval notation, then graph the solution set on the number line.

$1 - 3x > 14 - (4 - 6x)$ Interval Notation: _____

Graph:

$-\infty \longleftarrow\!\longrightarrow \infty$

5. Complete the table below.

Inequality	Graph	Interval Notation
$x < 0$	$-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$	
$-2 < x \le 1$	$-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$	
	$-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$	$[-3, \infty)$
	$-\infty \longleftarrow\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\longrightarrow \infty$ -3 -2 -1 0 1 2 3 4 5 6 7	