Unit 3: Solving Equations

KEY TERMS AND CONCEPTS	
Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms *in your own words*. Provide examples that are not identical to those in the Media Lesson.	
Equality	
Equivalence	
Algebraic Equation	
Solution to an Algebraic Equation	

Equivalent Equations	
Addition Property of Equality	
Subtraction Property of Equality	
Multiplication Property of Equality	
Division Property of Equality	
Solving an Equation	
Isolating the Variable	

Unit 3: Media Lesson

Section 3.1: Algebraic Equations

Definition
An <u>algebraic equation</u> is a mathematical sentence stating that an algebraic expression is *equal to* a specified value, variable, or another expression. The <u>solution</u> to an equation is the value, or values, that make the equation true.

Verify that a given value is a solution to an equation

Example 1: Verify that $x = $ -3 is a solution to the algebraic equation $5x - 2 = 8x + 7$.

Example 2: Is $m = $ -1 a solution to the algebraic equation $m + 9 = 3m + 5$?

Example 3: Is $a = 5$ a solution to the algebraic equation $-4(a + 1) = 6(1 - a)$?

Definition
<u>Equivalent equations</u> are two or more equations that have the same solution.

Example 4: Verify that $x = 2$ is a solution to the following equations.

$$8x - 5 = x + 9 \qquad\qquad 7x - 5 = 9 \qquad\qquad 7x = 14$$

Section 3.1 – YOU TRY

Complete the following problems. Show all steps as in the media examples.

a. Verify that $p = $ -9 is a solution to the algebraic equation $p - 4 = 2p + 5$.

b. Verify that $x = 2$ is a solution to the algebraic equation $2(5x - 12) = 1 - 5(x - 1)$.

Section 3.2: Equations and the Story of x

Definitions
• An algebraic **expression** is a mathematical statement that can contain numbers, variables, and operations (addition, subtraction, multiplication, division, etc…).
• An algebraic **equation** is a mathematical sentence stating that an algebraic expression is *equal to* a specified value, variable, or another expression.
• The **solution to an equation** is the value, or values, that make the equation true.

Example 1:   Tell the story of x in the expression $x + 7$, and use this to determine the
solution to the equation $x + 7 = 18$.

Story of x Solve   Check

Example 2:   Tell the story of x in the expression $4x$, and use this to determine the solution
to the equation $4x = 28$.

Story of x Solve   Check

Example 3:   Tell the story of x in the expression $5x + 1$, and use this to determine the
solution to the equation $5x + 1 = 21$.

Story of x Solve   Check

Example 4: Tell the story of x in the expression $2(x + 1) - 3$, and use this to determine the solution to the equation $2(x + 1) - 3 = 5$.

Story of x	Solve	Check

Example 5: Tell the story of x in the expression $\frac{3x-4}{2}$, and use this to determine the solution to the equation $\frac{3x-4}{2} = 7$.

Story of x	Solve	Check

Section 3.2 – YOU TRY

Tell the story of x in the expression $2x - 3$, and use this to determine the solution to the equation $2x - 3 = 15$. Write all steps of your solution process, as shown in the media examples.

Story of x	Solve	Check

Section 3.3: Solving One-Step Equations

Properties of Equality
The Addition/Subtraction Property of Equality: If $a = b$, then $a + c = b + c$.   If $a = b$, then $a - c = b - c$ The Multiplication/Division Property of Equality: If $a = b$, then $a \times c = b \times c$.   If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$

Definition
To <u>solve an equation</u> means to "undo" all the operations of the equation, leaving the variable by itself on one side. This is known as **isolating the variable.**

Solve for the variable in each of the following equations. Check your answers.

Example 1: $x + 7 = 18$ **Example 2:** $r - 4 = -5$

Example 3: $-4 + b = 45$ **Example 4:** $3 = 19 + m$

Example 5: $-3y = -42$ **Example 6:** $\frac{x}{6} = -5$

Example 7: $\frac{3}{4}a = 8$

Example 8: $17 = -x$

Section 3.3 – YOU TRY

Solve for the variable in each equation and check your answer. Show all steps as in the media examples.

a. $12 + x = -40$

b. $\frac{3}{5}n = -2$

c. $14 = -x$

d. $-3 = \frac{w}{5}$

Section 3.4: Solving Two-Step Equations

STEPS FOR SOLVING A LINEAR TWO-STEP EQUATION

1. Apply the Addition/Subtraction Property of Equality.

2. Apply the Multiplication/Division Property of Equality to isolate the variable.

3. Check by substituting your answer into the original equation.

Solve for the variable in each of the following equations. Check your answers.

Example 1:   Solve: $2b - 4 = 12$ Check:

Example 2:   Solve: $4 + 3r = 5$ Check:

Example 3:   Solve: $3 = 19 - 2m$ Check:

Example 4:   Solve: $11 - y = 32$ Check:

Example 5: Solve: $3 + \frac{3}{5}x = 12$ Check:

Section 3.4 – YOU TRY

Solve for the variable in each equation and check your answer. Show all steps as in the media examples.

    a. Solve: $14 - 3x = -40$ Check:

    b. Solve: $\frac{3}{4}w - 8 = -2$ Check:

    c. Solve: $14 = 2 - x$ Check:

Section 3.5: Solving Multi-Step Equations

STEPS FOR SOLVING A LINEAR EQUATION
1. Simplify each side of the equation. Remove parenthesis if necessary. Combine like terms.
2. Add or subtract terms on each side of the equation so that all terms containing the variable are on one side and all constant terms are on the other side.
3. Simplify each side of the equation by combining like terms.
4. Apply the Multiplication/Division Property of Equality to isolate the variable.
5. Check by substituting the solution into the original equation.

Solve for the variable in each of the following equations. Check your answers.

Example 1: Solve $x - 5 = 4x + 7$ Check

Example 2: Solve $3(4n - 2) = 5(n + 3)$ Check

Example 3: Solve $4 - (2y - 1) = 2(5y + 9) + y$ Check:

Section 3.5 – You Try

Solve for the variable in each equation and check your answer. Show all steps as in the media examples.

a. Solve $m - 5 = 8m + 2$ Check:

b. Solve $2(5x - 12) = -(5x - 6)$ Check:

Section 3.6: Solving Equations – Applications

> For this type of problem, first determine the Givens and the Goal, then form a Strategy,
> Solve, and Check. Write your answer in a complete sentence.

Example 1:   The maximum heart rate is the highest heart rate achieved during maximal exercise. In general, you gain the most benefits and lessen the risks when you exercise within your *target* heart rate zone. Usually this is when your exercise heart rate (pulse) is about 70% percent of your maximum heart rate. The formula $T = 0.7(220 - a)$, gives the target heart rate, *T*, in beats per minute, for a person who is *a* years of age.   Determine the age of a person whose target heart rate is 135 beats per minute.

GIVEN: GOAL:

STRATEGY:

SOLUTION: CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

Section 3.6 – YOU TRY

For this problem, identify the Givens the Goal. Form a strategy, solve, check, and write your answer in a complete sentence. Show all steps.

The cost of tuition at a local community college is given by the equation $C = 76n$, where C represents the total cost of tuition and n represents the number of credits taken. If you have $800 dollars to spend on tuition, how many credits can you take?

GIVEN: GOAL:

STRATEGY:

SOLUTION: CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

Section 3.7: Writing Equations

Steps for Writing and Solving Equations
Step 1: Read and understand the problem. Underline the givens and circle the goal.
Step 2: Form a strategy to solve the problem.
Step 3: Choose a variable to represent the unknown quantity.
Step 4: Read every word in the problem, and translate the given information into an algebraic equation.
Step 5: Solve the equation
Step 6: Write your answer in a complete sentence

Example 1: The cost of leasing a new Ford mustang is $2,311 for a down payment and processing fee plus $276 per month. For how many months can you lease this car with $10,000?

Example 2: You have just bought a new Sony 55" 3D television set for $1,600. The value of the television set decreases by $250 per year. How long before the television set is worth half of its original value?

Section 3.7 – YOU TRY

✏️ Your yard is a mess, and you decide to hire a landscaper. The Garden Pros charges a $50 consultation fee plus $36 per hour for the actual work.   If the total cost is $212, how many hours did the landscapers work?

a. Write an equation to represent this situation. Clearly indicate what the variable represents.

b. Solve the equation. Show all work, and write your answer in a complete sentence. Your answer must include correct units of measure.

Unit 3: Practice Problems

1. Verify that $a = -1$ is a solution to $4 - a = 6a + 11$. Show all work.

2. Verify that $x = -5$ is a solution to $3(2x + 4) = 8(x + 2) + 6$. Show all work.

3. Is $x = 8$ a solution to the equation $-16 = \frac{3}{4}x - 10$? Answer yes or no, and show all supporting work.

4. Is $x = -3$ a solution to the equation $3(6 + 2x) = 8 + (x - 5)$? Answer yes or no, and show all supporting work.

5. Tell the story of x in the expression $5x + 7$, and use this to determine the solution to the equation $5x + 7 = 62$.

Story of x	Solve	Check

6. Tell the story of x in the expression $4(x - 8) + 2$, and use this to determine the solution to the equation $4(x - 8) + 2 = 22$.

Story of x	Solve	Check

7. Tell the story of x in the expression $\frac{4x-2}{3}$, and use this to determine the solution to the equation $\frac{4x-2}{3} = 10$.

Story of x	Solve	Check

8. Solve for the variable in each of the following equations. Reduce, simplify, and check your
   answers. Show all steps, and box your answer.

   a. $8x - 2 = 30$ Check:

   b. $5 - x = 3$ Check:

   c. $-\frac{1}{2}x - 4 = 8$ Check:

   d. $\frac{2}{3}x + 3 = 15$ Check:

e. $4x - 8 = -x + 7$ Check:

f. $\frac{3}{4}x - \frac{1}{2} = \frac{9}{8}x + \frac{3}{2}$ Check:

g. $6x - 4(-2x + 8) = 10$ Check:

h. $-2(4x - 2) = -(2x - 8)$ Check:

i. $(2x - 7) - (4x + 8) = 4(x + 6)$ Check:

Applications

For each of the following, *underline* the Givens and *circle* the Goal of the problem. Form a Strategy, Solve, and Check. Show all work, and write your answers in complete sentences.

9. John is a door to door vacuum salesman. His weekly salary, S, is $200 plus $50 for each vacuum he sells. This can be written as S = 200 + 50v, where v is the number of vacuums sold. If John earns $1000 for a week's work, how many vacuums did he sell?

10. Paul is planning to sell bottled water at the local Lollapalooza. He buys 2 crates of water (2000 bottles) for $360 and plans on selling the bottles for $1.50 each. Paul's profit, P in dollars, from selling b bottles of water is given by the formula $P = 1.5b - 360$. How many bottles does Paul need to sell in order to break even?

11. Ringo has $100 in the bank and is adding $50 each week in savings. George has $250 in the bank, and is adding $40 each week in savings. Their plan is to wait until their savings are equal and then buy a Magic Yellow Bus and take a road trip. They figure out that the equation can be written as $50w + 100 = 40w + 250$, where w is the number of weeks. How long will it take for their savings to be equal?

12. The formula for the area, A, of a triangle with base b and height h is $A = \frac{1}{2}bh$. Determine the height of a triangle with a base of 18 inches and area 84.6 square inches. Round your answer to the nearest tenth, and include appropriate units in your answer.

13. Suppose you want to accumulate $1,000,000 for your retirement in 30 years. You decide to put money into an account that earns 3% interest compounded annually. How much should you deposit?   The formula for compound interest is $A = P(1 + r)^t$, where A is the accrued amount after t years, P is the starting principal, and r is the annual interest rate expressed as a decimal. Round your answer *up* to the nearest cent.

14. Andrew and Andrea want to start a college fund for their baby girl. They decide to put money into an investment that is expected to earn 4.2% simple interest each year. How much would they have to deposit now in order to accumulate $100,000 by the time their newborn goes to college in 18 years?   The formula for simple interest is $A = P + Prt$, where A is the accrued value of the investment after t years, r is the interest rate (expressed as a decimal), and P is the starting principal invested. Round your answer *up* to the nearest cent.

15. February is a busy time at Charlie's Chocolate Shoppe! During the week before Valentine's Day, Charlie advertises that his chocolates will be selling for $1.80 a piece (instead of the usual $2.00 each). The fixed costs to run the Chocolate Shoppe total $450 for the week, and he estimates that each chocolate costs about $0.60 to produce. Write an equation to represent Charlie's **profit**, P, from selling n chocolates during the week before Valentine's Day. (HINT: Profit = Revenue – Total Costs) use this equation the number of Chocolates Charlie will need to sell in order to break even.

16. A new Sony 55" 3D television set costs $2,499. You are going to pay $600 as a down payment, and pay the rest in equal monthly installments for one year. Write an equation to represent this situation, and use it to determine how much you should pay each month. Clearly indicate what the variable in your equation represents.

17. Your yard is a mess, and you decide to hire a landscaper. The Greenhouse charges a $20 consultation fee plus $11 per hour for the actual work. Garden Pros does not charge a consulting fee, but charges $15 per hour for the actual work. Write an equation that will help you determine the number of hours at which the two companies charge the same. Clearly indicate what the variable represents. Solve the equation, and write your answer in a complete sentence.

18. Let p represent the marked price of an item at Toys R Us. Emma's aunt gave her a $50 gift card to Toys R Us for her birthday. If sales tax is currently 9%, set up an equation to express how much she can spend using her gift card. Solve the equation, and interpret your answer in a complete sentence.

Unit 3: Solving Equations   Practice Problems

Extension

19. Solve for the variable in each of the following equations. Reduce, simplify, and check your answers. Show all steps, and box your answer.

    a. $2(4x + 3) = 8x + 1$ b. $5(x + 6) - x = 4(x + 7) + 2$

20. Solve the following nonlinear equations.

    a. $x^2 = 25$ b. $x^3 = 27$ c. $|x| = 3$

    d. $\sqrt{x} = 7$ e. $\sqrt[3]{x} = 2$ f. $\frac{1}{x} = 4$

21. Write a story problem for the equation shown below. Solve the problem, and write your answer in a complete sentence.
$$300 - 50x = 0$$

Unit 3: Review

1. Solve the following equations for x. Show your work. Reduce, simplify and CHECK your answers!

   a. $7 - (a - 3) = 3(2a - 6)$ Check

   b. $-31 = \dfrac{3}{5}x - 10$ Check

2. The formula to convert from Celsius to Fahrenheit is $F = \dfrac{9}{5}C + 32$. The temperature on a summer day in Phoenix, Arizona is 113°F. What would this temperature be in degrees Celsius? Show all work, and write your answer in a complete sentence

3. You decide to invest $7000 into an account that pays 5% simple interest each year. How long
     will it take for the investment to double in value?

     The formula for simple interest is $A = P + Prt$, where A is the accrued value of the
     investment after t years, r is the interest rate (expressed as a decimal), and P is the starting
     principal invested.

     Show all steps, and write your answer in a complete sentence.

4. Carlos recently hired a roofer to do some necessary work. On the final bill, Carlos was
     charged a total of $1105. $435 was listed for parts and the rest for labor. If the hourly rate
     for labor was $67, how many hours of labor was needed to complete the job?

     a. Write an *equation* that can be used to determine the number of hours needed to complete
          the job. **Clearly indicate what the variable represents.**

     b. Solve the equation. Show all steps, and write your answer in a complete sentence.